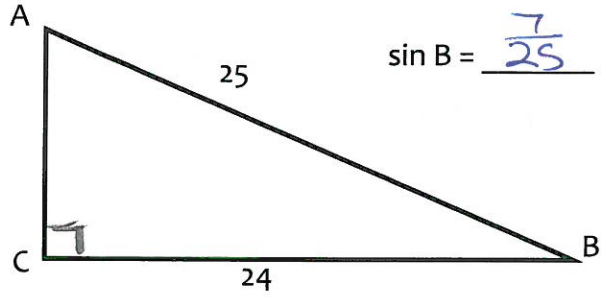
Name _____
Date _____

Trigonometry
Midterm Review Questions

1. Find the following ratios.



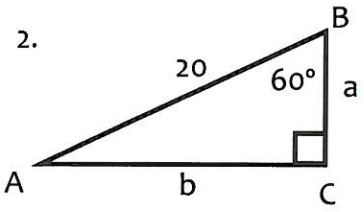
7

sin A = $\frac{24}{25}$ cos A = $\frac{7}{25}$ tan A = $\frac{24}{7}$

sin B = $\frac{7}{25}$ cos B = $\frac{24}{25}$ tan B = $\frac{7}{24}$

Solve each triangle.

2.

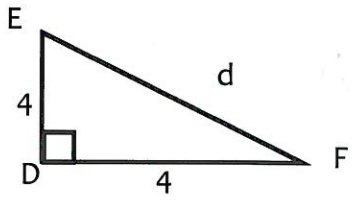


∠A = 30°

side a = 10

side b = $10\sqrt{3}$

3.



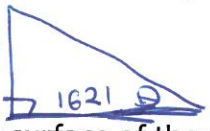
∠E = 45°

∠F = 45°

side d = $4\sqrt{2}$

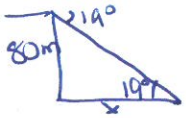
4. The Eiffel Tower is 1063 feet tall. A man is standing 1621 feet away from the center of the base of the tower. What is the angle of elevation?

$\tan\theta = \frac{1063}{1621}$ $\theta = 33.26°$



$\tan^{-1}\left(\frac{1063}{1621}\right) = \theta$

5. From the top of a vertical cliff, 80 meters above the surface of the ocean, the measure of the angle of depression to a marker on the surface of the ocean is 19°. How far is the marker from the foot of the cliff?



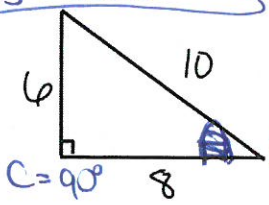
$\tan 19° = \frac{80}{x}$ $x = \frac{80}{\tan(19°)}$ $x = 232.34$ m

6. Use right triangle trigonometry to solve the triangle:

$B = 53.13°$

$\tan A = \frac{6}{8}$

$\tan^{-1}\left(\frac{6}{8}\right) = A$



Name _____
Date _____

Trigonometry
Midterm Review Questions

7. Sketch the arc and state the six trig. values.

a. $\frac{19\pi}{4}$

$\sin\left(\frac{19\pi}{4}\right) = \frac{1}{\sqrt{2}}$ $\quad \csc\left(\frac{19\pi}{4}\right) = \sqrt{2}$

$\cos\left(\frac{19\pi}{4}\right) = \frac{1}{\sqrt{2}}$ $\quad \sec\left(\frac{19\pi}{4}\right) = -\sqrt{2}$

$\tan\left(\frac{19\pi}{4}\right) = -1$ $\quad \cot\left(\frac{19\pi}{4}\right) = -1$

b. -7π

$\sin(-7\pi) = 0$ $\quad \csc(-7\pi) = \text{undef}$

$\cos(-7\pi) = -1$ $\quad \sec(-7\pi) = -1$

$\tan(-7\pi) = 0$ $\quad \cot(-7\pi) = \text{undef}$

8. Sketch the following angles and state the six trig values:

a. 390°

$\sin(390^\circ) = \frac{1}{2}$ $\quad \csc(390^\circ) = 2$

$\cos(390^\circ) = \frac{\sqrt{3}}{2}$ $\quad \sec(390^\circ) = \frac{2}{\sqrt{3}}$

$\tan(390^\circ) = \frac{1}{\sqrt{3}}$ $\quad \cot(390^\circ) = \sqrt{3}$

b. -405°

$\sin(-405^\circ) = -\frac{1}{\sqrt{2}}$ $\quad \csc(-405^\circ) = -\sqrt{2}$

$\cos(-405^\circ) = \frac{1}{\sqrt{2}}$ $\quad \sec(-405^\circ) = \sqrt{2}$

$\tan(-405^\circ) = -1$ $\quad \cot(-405^\circ) = -1$

9. Find two coterminal angles (one positive and one negative) for the following:

a. $\frac{3\pi}{5}$ $\quad -\frac{7\pi}{5}, \frac{13\pi}{5}$

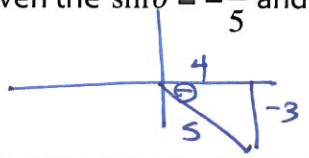
b. 76° $\quad -284^\circ, 436^\circ$

10. Convert the following from radians to degrees or degrees to radians:

a. $\frac{2\pi}{9}$ $\quad \left(\frac{2\pi}{9}\right)\left(\frac{180^\circ}{\pi}\right) = 40^\circ$

b. 124° $\quad 124^\circ\left(\frac{\pi}{180^\circ}\right) = \frac{31\pi}{45}$

11. Given the $\sin\theta = -\frac{3}{5}$ and θ is in the fourth quadrant. Find all six trigonometric values.



$\sin\theta = -\frac{3}{5}$ $\quad \csc\theta = -5/3$

$\cos\theta = 4/5$ $\quad \sec\theta = 5/4$

$\tan\theta = -3/4$ $\quad \cot\theta = -4/3$

12. Find the linear velocity, in inches per minute, of the tip of an hour hand of a clock, if the hour hand has a length of 1 ft.

$v = st = \frac{r\theta}{t} = \frac{1\text{ ft}(2\pi)}{12\text{ hrs}} = \left(\frac{\pi\text{ ft}}{6\text{ hr}}\right)\left(\frac{12\text{ in}}{1\text{ ft}}\right)\left(\frac{1\text{ hr}}{60\text{ mins}}\right) = .10\text{ in/min}$

13. A race car driver moves around a curved track at a speed of 160 mph. If the radius of the circle, of which the driver's path is an arc, is 1,500 ft., what is the driver's angular velocity in radians per minute around the curve?

9.39 rads/min

~~.55~~ rads/hr

$\omega = \frac{\theta}{t}$ $\quad$ So $v = r\omega$

$160\text{ mph} = (1500\text{ ft}\ \omega)\left(\frac{1\text{ mile}}{5280\text{ ft}}\right)$

$\frac{160\text{ miles}}{\text{hr}} = \left(\frac{1500}{5280}\text{ miles}\right)\omega = \left(\frac{5280\text{ rads}}{\text{hr}}\right)\left(\frac{1\text{ hr}}{60\text{ mins}}\right)$

Name _____

Date _____

Trigonometry
Midterm Review Questions

14 a. If $\cos s = -\frac{3}{5}$ and sin<0, find tan s.

$\tan s = \frac{4}{3}$

b. If $\tan s = -1$ and sin>0, find cos s.

$\cos s = \frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$

c. If sec s = -2 and sin>0, find tan s.

$\cos s = \frac{-1}{2}$

$\tan s = \frac{\sqrt{3}}{-1} = -\sqrt{3}$

15. Complete the Table

Equation	Amplitude	Period	Phase Shift	Vertical Shift
a. $y = -2\cos(3x)$	2	$\frac{2\pi}{3}$	—	—
b. $y = \frac{1}{2}\sin(x) - 3$	$\frac{1}{2}$	2π	—	down 3
c. $y = \tan(2x - \frac{\pi}{2})$	1	$\frac{\pi}{2}$	rt $\frac{\pi}{4}$	—
d. $y = \cos(\frac{1}{2}x - \frac{\pi}{4}) + 1$	1	4π	rt $\frac{\pi}{2}$	up 1

reflect

$\tan 2(x - \frac{\pi}{4})$

$\cos(\frac{1}{2}(x - \frac{\pi}{2})) + 1$

16. $y = 2\sin(3x - \pi)$

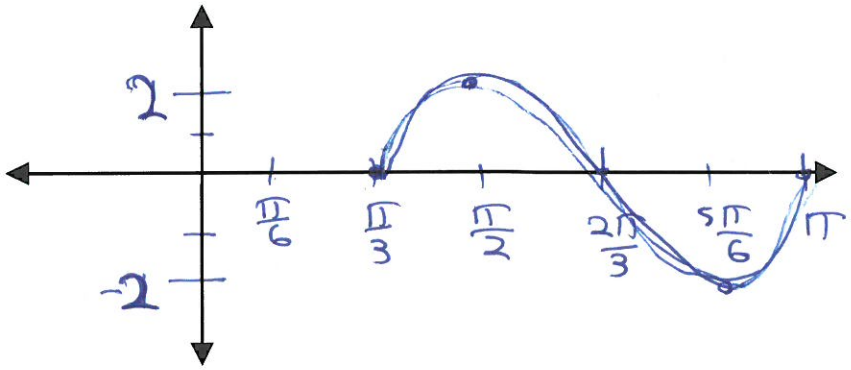
$y = 2\sin 3(x - \frac{\pi}{3})$

A: 2

P: $\frac{2\pi}{3}$

PS: rt $\frac{\pi}{3}$

VS: —



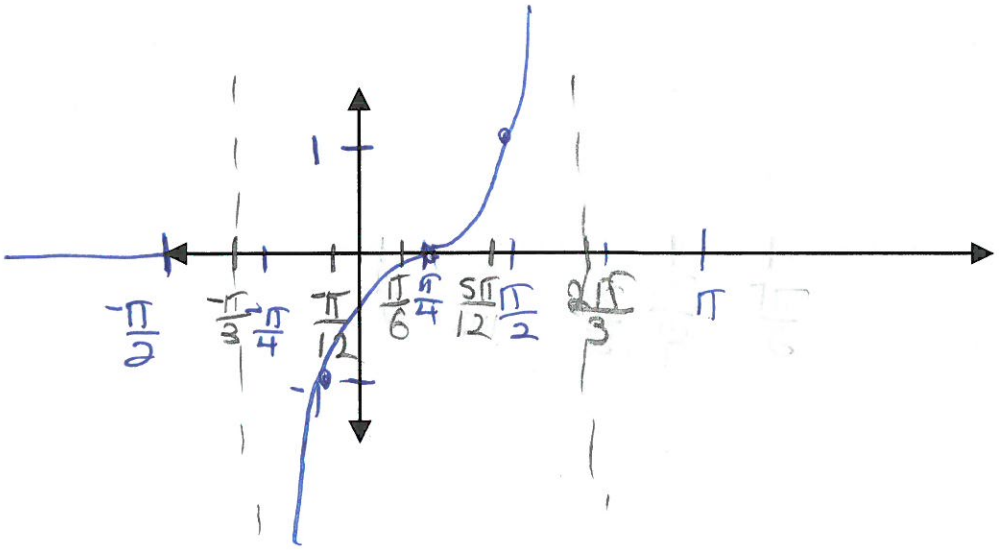
17. $y = \tan(x - \frac{\pi}{6})$

A: 1

P: π

PS: rt $\frac{\pi}{6}$

VS: —



Name _____
Date _____

Trigonometry
Midterm Review Questions

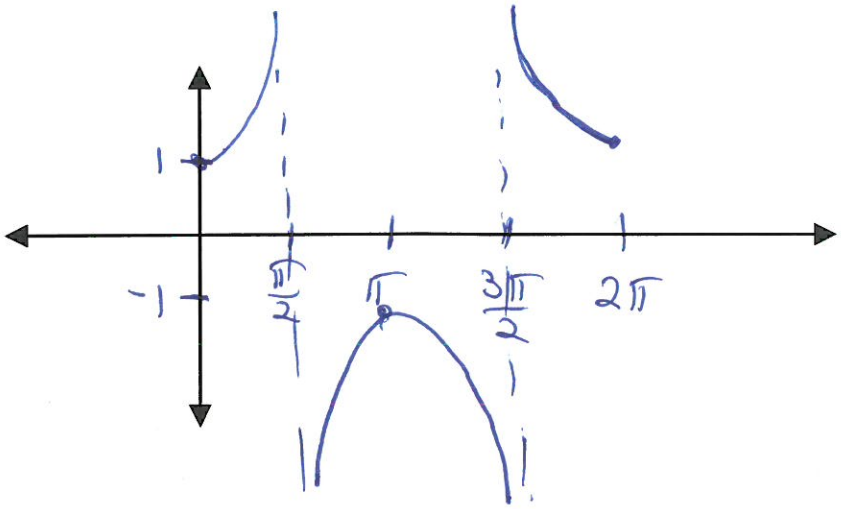
18. $y = \sec x - 2$

A: 1

P: 2π

PS: —

VS: down 1



Given one period of a trigonometric function, write the equation of the function.

19. reflect

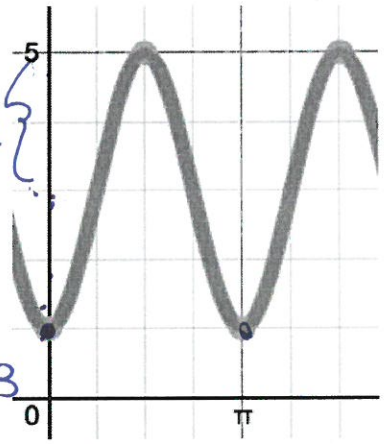
P = π

VS +3

PS —

A = 2

$y = -2\cos 2(x) + 3$



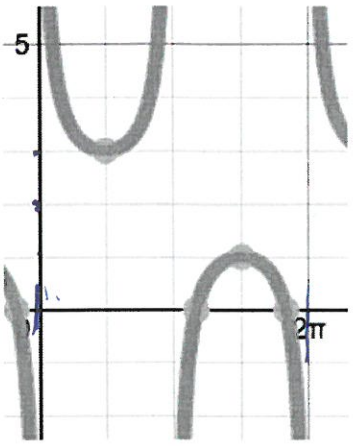
20.

A = 1

P = 2π

VS = +2

PS = —



$y = \sec x + 2$

21.

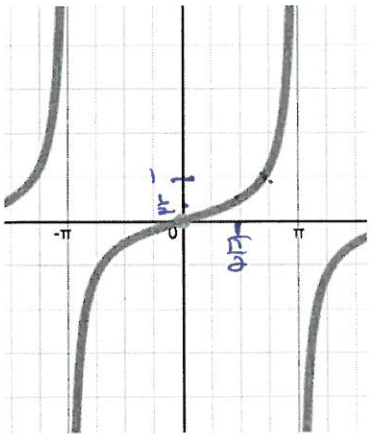
P = 2π

PS —

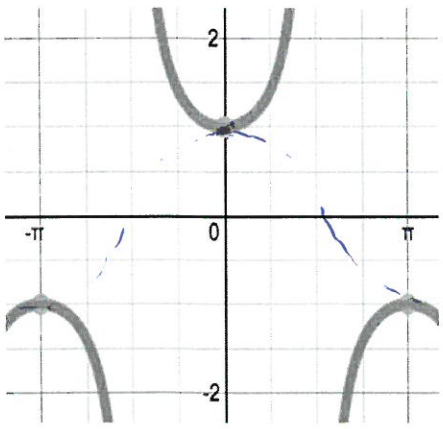
VS —

A = $\frac{1}{2}$

$y = \frac{1}{2}\tan\frac{1}{2}(x)$



22.



A = 1

P = 2π

PS —

VS —

$y = \sec x$

Name _____ Trigonometry
Date _____ Midterm Review Questions

Answer the following questions

23. If a function has a period of $\frac{2\pi}{3}$ and one cycle begins at $\frac{\pi}{2}$ where will that cycle end?

$\frac{2\pi}{3} + \frac{\pi}{2} = \frac{7\pi}{6}$

24. If a sine curve was shifted $\frac{\pi}{4}$ to the left, has an amplitude of 3, and a period of π, what is the equation?

$-\frac{\pi}{4}$ $y = 3\sin 2\left(x + \frac{\pi}{4}\right)$

Sine Regression

25. The table below lists the average monthly temperatures in degrees Fahrenheit for the city of Baltimore, MD. The table gives y, the average monthly temperature, as a function of t, the month, where t = 0 indicates January.

Table: *Average monthly temperature for Baltimore, MD*

	Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec
t	0	1	2	3	4	5	6	7	8	9	10	11
y	31.8	34.8	44.1	53.4	63.4	72.5	77	75.6	68.5	56.6	46.8	36.7

a. By hand, find the amplitude of the data. $\frac{77 - 31.8}{2} = 22.6$

b. By hand, find the vertical shift of the function. $\frac{77 + 31.8}{2} = 54.4$

c. Use your calculator to find the sine function that models this data (round to the nearest hundredths place)

y ~~$= 22.46 \sin(.51x - 1.55) + 54.35$~~

$y = 22.46 \sin.51(x - 3.04) + 54.35$

d. What will the temperature be the following June (from the start of the data)?

$22.46 \sin(.51(5) - 1.55) + 54.35 = 54.74$

e. What is the period of this function? 12.31

f. Why does the period make sense for our set of data (in context)?

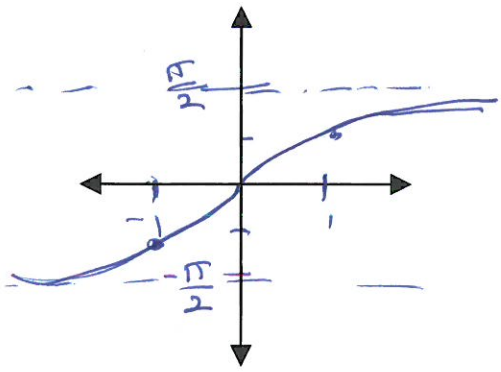
12 months in a year

Name _____ Trigonometry
Date _____ Midterm Review Questions

Be able to **Graph** each inverse function given. (It may be helpful for you to fill in the table first.) Then **identify the domain, the range and the y-intercept.**

26. $y = \arctan x$

X	Y
—	$-\frac{\pi}{2}$
-1.73	$-\frac{\pi}{3}$
-1	$-\frac{\pi}{4}$
-.58	$-\frac{\pi}{6}$
0	0
.58	$\frac{\pi}{6}$
1	$\frac{\pi}{4}$
1.73	$\frac{\pi}{3}$
—	$\frac{\pi}{2}$



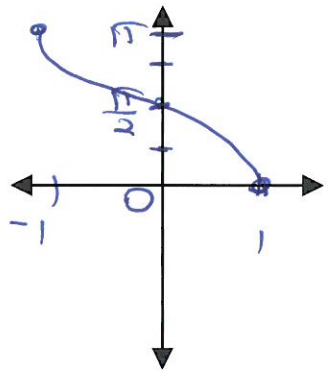
Domain: $\mathbb{R}$

Range: $-\frac{\pi}{2} < y < \frac{\pi}{2}$

Y-intercept: $(0,0)$

27. $y = \cos^{-1} x$

X	Y
1	0
$\frac{\sqrt{3}}{2}$ (.87)	$\frac{\pi}{6}$
$\frac{\sqrt{2}}{2}$ (.71)	$\frac{\pi}{4}$
$\frac{1}{2}$ (.5)	$\frac{\pi}{3}$
0	$\frac{\pi}{2}$
-.5	$\frac{2\pi}{3}$
-.71	$\frac{3\pi}{4}$
-.87	$\frac{5\pi}{6}$
-1	π



Domain: $-1 \le x \le 1$

Range: $0 \le y \le \pi$

Y-intercept: $(0, \frac{\pi}{2})$

Find the exact values!

28. $\arctan(-1) =$ $-\frac{\pi}{4}$

29. $\tan^{-1}(\sqrt{3})$ $\frac{\pi}{3}$

30. $\cos^{-1}\left(-\frac{1}{2}\right)$ $\frac{2\pi}{3}$

31. $\arcsin\left(\frac{\sqrt{3}}{2}\right)$ $\frac{\pi}{3}$

32. $\sin^{-1}(0)$ 0

33. $\arccos\left(-\frac{\sqrt{3}}{2}\right)$ $\frac{5\pi}{6}$

Name _____
Date _____

Trigonometry
Midterm Review Questions

Identify the domain and range for each of the inverse reciprocal functions.

34. $y = \sec^{-1}(x)$ Domain: $-\infty < x \le -1 \cup 1 \le x < \infty$

Range: $0 \le y \le \pi$

35. $y = \csc^{-1}(x)$ Domain: $-\infty < x \le -1 \cup 1 \le x < \infty$

Range: $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

36. $y = \cot^{-1}(x)$ Domain: $\mathbb{R}$

Range: $0 < y < \pi$

Find the exact values!

37. $\text{arc}\csc(-2)$

$\csc\theta = -2$

$\sin\theta = -\frac{1}{2}$

$\theta = -\frac{\pi}{6}$

38. $\sec^{-1}(-\sqrt{2})$

$\cos\theta = \frac{-1}{\sqrt{2}}$

$\theta = \frac{3\pi}{4}$

39. $\csc^{-1}\left(-\frac{1}{2}\right)$

$\sin\theta = -2$

No solution!!

40. $\text{arc}\csc\left(\frac{-2\sqrt{3}}{3}\right)$

$\sin\theta = \frac{-3}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$\sin\theta = \frac{-3\sqrt{3}}{6} = \frac{-\sqrt{3}}{2}$

$\theta = -\frac{\pi}{3}$

41. $\sec^{-1}(\text{undefined})$

$\cos\theta = 0$

$\theta = \frac{\pi}{2}$

42. $\text{arc}\cot(\sqrt{3})$

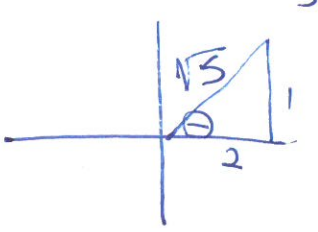
$\tan\theta = \frac{1}{\sqrt{3}}$

$\theta = \frac{\pi}{6}$

43. $\sin\left(\arctan\frac{1}{2}\right)$

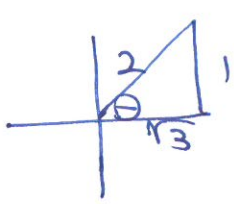
$\sin(\theta) = \frac{1}{\sqrt{5}}$ or $\frac{\sqrt{5}}{5}$

$\tan\theta = \frac{1}{2}$



44. $\tan\left(\cos^{-1}\frac{\sqrt{3}}{2}\right)$

$\tan\theta = \frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{3}$



45. $\csc^{-1}(\cos\pi)$

$\csc^{-1}(-1) =$

$\sin\theta = -1$

$-\frac{\pi}{2}$

$\frac{3\pi}{2}$ not in range

Name _____ Trigonometry
Date _____ Midterm Review Questions

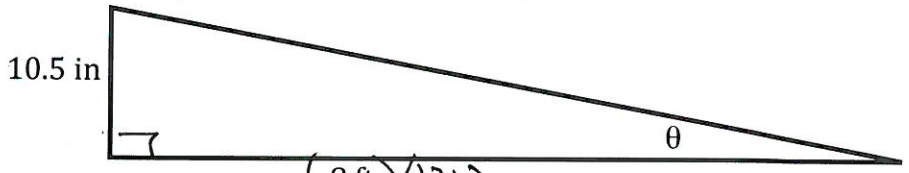
46. The formula $D = 24\left[1 - \frac{\cos^{-1}(\tan i \tan\theta)}{\pi}\right]$ is used to approximate the number of hours of daylight when the angle of declination of the sun is i° at a location θ° north of the equator. How many hours of daylight are predicted in Anchorage Alaska $(\theta = 66^\circ 30')$ on July 4th $(i = 22^\circ 48')$?

$\theta = 66.5^\circ = (66.5)\left(\frac{\pi}{180}\right) = \frac{133\pi}{360}$

$i = 22.8^\circ = (22.8)\left(\frac{\pi}{180}\right) = \frac{19\pi}{150}$

$D = 24\left[1 - \frac{\cos^{-1}\left[\tan\left(\frac{133\pi}{360}\right)\tan\left(\frac{19\pi}{150}\right)\right]}{\pi}\right] = 22.02$ hours

47. Mr. Izzo has decided to expand his tree house. It will be 8 feet long now (instead of 6 feet long.) To ensure the structure does not collapse due to snow building up on the roof, Mr. Izzo has lifted the roof another 3.5 inches making it 10.5 inches. What is the angle of elevation?

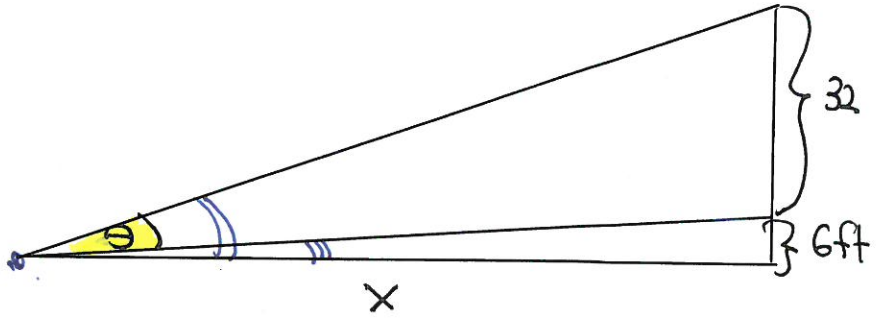


$(8\text{ ft})\left(\frac{12\text{ in}}{1\text{ ft}}\right) = 96$ in

$\tan\theta = \frac{10.5}{96}$

$\theta = \tan^{-1}\left(\frac{10.5}{96}\right) = 6.24^\circ$

48. A movie theater recently advertised that they were buying a new screen that is 32 feet tall. When you sit down, the bottom of the screen is 6ft. above your eye level. The angle formed by drawing a line from your eye to the bottom of the screen and your eye and the top of the screen is called the **viewing angle**. In the figure, θ is the viewing angle. Suppose that you sit x feet from the screen. Write a function that represents this viewing angle.



$\theta = \tan^{-1}\left(\frac{38}{x}\right) - \tan^{-1}\left(\frac{6}{x}\right)$

Name _____
Date _____

Trigonometry
Midterm Review Questions

Find the General Solution.

49. $\sin\theta = \frac{-\sqrt{3}}{2}$

$\theta = \frac{5\pi}{3} + 2\pi k, \frac{4\pi}{3} + 2\pi k$

Find the solution on the interval $0 \le \theta < 2\pi$.

50. $\sec^2\theta = 4$

$\sec\theta = \pm 2$

$\cos\theta = \pm\frac{1}{2}$

$\theta = \frac{\pi}{3} + 2\pi k, \frac{2\pi}{3} + 2\pi k, \frac{4\pi}{3} + 2\pi k, \frac{5\pi}{3} + 2\pi k$

51. $\tan x + \sqrt{3} = 0$

$\tan x = -\sqrt{3}$

$x = \frac{2\pi}{3}, \frac{5\pi}{3}$

52. $\csc^2\theta - 4 = 0$

$\csc^2\theta = 4$

$\sin^2\theta = \frac{1}{4}$

$\sin\theta = \pm\frac{1}{2}$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

53. $4\sin^2 x = 1 + 4\cos x$

$4(1-\cos^2 x) = 1 + 4\cos x$

$4 - 4\cos^2 x = 1 + 4\cos x$

$4\cos^2 x + 4\cos x - 3 = 0$

$(2\cos x - 1)(2\cos x + 3) = 0$

$2\cos x = 1$ $\quad \cos x = -\frac{3}{2}$ (undef)

$\cos x = \frac{1}{2}$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$

54. $(\tan\theta - 1)(\sec\theta - 1) = 0$

$\tan\theta = 1$ $\quad \sec\theta = 1$

$\theta = \frac{\pi}{4}, \frac{5\pi}{4}$ $\quad \cos\theta = 1$

$\theta = 0$

Simplify to **one** term.

11. $\csc\theta\tan\theta$

$\frac{1}{\sin\theta}\cdot\frac{\sin\theta}{\cos\theta} = \frac{1}{\cos\theta} = \sec\theta$

12. $\frac{\sec\theta}{\tan\theta}$

$= \frac{\frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}} = \frac{1}{\sin\theta} = \csc\theta$

Prove the following identities

55. $\frac{1+\tan\theta}{1-\tan\theta} = \frac{1+\cot\theta}{-1+\cot\theta}$

$\frac{1+\frac{\sin\theta}{\cos\theta}}{1-\frac{\sin\theta}{\cos\theta}} =$

$\frac{\frac{\cos\theta+\sin\theta}{\cos\theta}}{\frac{\cos\theta-\sin\theta}{\cos\theta}} =$

$\frac{\frac{\cos\theta}{\sin\theta}+\frac{\sin\theta}{\sin\theta}}{\frac{\cos\theta}{\sin\theta}-\frac{\sin\theta}{\sin\theta}} =$

$\frac{\cot\theta+1}{\cot\theta-1} = \frac{1+\cot\theta}{-1+\cot\theta}$

56. $1 - \frac{\cos^2\theta}{1+\sin\theta} = \sin\theta$

$\frac{1+\sin\theta}{1+\sin\theta} - \frac{\cos^2\theta}{1+\sin\theta} =$

$\frac{1+\sin\theta-\cos^2\theta}{1+\sin\theta} =$

$\frac{1+\sin\theta-(1-\sin^2\theta)}{1+\sin\theta} =$

$\frac{\sin^2\theta+\sin\theta}{1+\sin\theta}$

$\frac{\sin\theta(\sin\theta+1)}{1+\sin\theta} = \sin\theta$

Name _____
Date _____

Trigonometry
Midterm Review Questions

Prove the following identities

57. $\dfrac{\sin x}{1+\cos x}+\dfrac{1+\cos x}{\sin x}=2\csc x$

$\dfrac{\sin^2 x}{\sin x(1+\cos x)}+\dfrac{\sin x(1+\cos x)^2}{\sin x(1+\cos x)}=$

$\dfrac{\sin^2 x+(1+2\cos x+\cos^2 x)}{\sin x(1+\cos x)}=$

$\dfrac{\sin^2 x+\cos^2 x+1+2\cos x}{\sin x(1+\cos x)}=$

$\dfrac{2+2\cos x}{\sin x(1+\cos x)}=\dfrac{2(1+\cos x)}{\sin x(1+\cos x)}=$

$\dfrac{2}{\sin x}=2\csc x$

58. $\tan\theta\cot\theta-\sin^2\theta=\cos^2\theta$

$\tan\theta\left(\dfrac{1}{\tan\theta}\right)-(1-\cos^2\theta)=$

$1-1+\cos^2\theta=$

$\cos^2\theta=\cos^2\theta$ ✓

59. $\dfrac{\sec\theta}{\csc\theta}+\dfrac{\sin\theta}{\cos\theta}=2\tan\theta$

$\dfrac{\frac{1}{\cos\theta}}{\frac{1}{\sin\theta}}+\dfrac{\sin\theta}{\cos\theta}$

$\dfrac{\sin\theta}{\cos\theta}+\dfrac{\sin\theta}{\cos\theta}$

$\tan\theta+\tan\theta$

$2\tan\theta=2\tan\theta$ ✓

60. $\dfrac{\csc\theta-1}{\cot\theta}=\dfrac{\cot\theta}{\csc\theta+1}$

$\dfrac{(\csc\theta-1)(\csc\theta+1)}{\cot\theta(\csc\theta+1)}=\dfrac{\cot\theta}{(\csc\theta+1)}$

$\dfrac{\csc^2\theta-1}{\cot\theta(\csc\theta+1)}=$

$\dfrac{\cot^2\theta}{\cot\theta(\csc\theta+1)}=$

$\dfrac{\cot\theta}{\csc\theta+1}=\dfrac{\cot\theta}{\csc\theta+1}$