Algebra II and Algebra II GT

Final Exam Review

2014-2015

Quadratics

1. Solve for x: $2x^2 - 46x + 252 = 0$

$2(x^2 - 23x + 126) = 0$

$x^2 - 23x + 126 = 0$

$(x-14)(x-9) = 0$

$x = 9, 14$

2. Solve for x: $x^2 - 4x - 6 = 0$

$x = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(-6)}}{2(1)} = \frac{4 \pm \sqrt{40}}{2} = \frac{4 \pm 2\sqrt{10}}{2} = 2 \pm \sqrt{10}$

3. Solve for x: $x^2 - 6x = 0$

$x(x-6) = 0$

$x = 0 \quad x = 6$

4. Solve for x: $3(x-2)^2 - 16 = 0$

$3(x-2)^2 = 16$

$(x-2)^2 = 16/3$

$x - 2 = \pm\sqrt{\frac{16}{3}}$

$x - 2 = \pm\frac{4\sqrt{3}}{3}$

$x = 2 \pm \frac{4\sqrt{3}}{3}$

5. State the number of real roots for the equation $x^2 - 18x + 81 = 0$

$b^2 - 4ac = (-18)^2 - 4(1)(81) = 0$

There is one real solution.

6. State the nature of the roots for the equation $x^2 + 3x + 8 = 0$

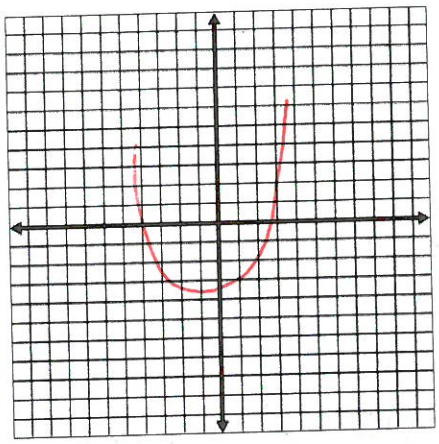
$b^2 - 4ac$

$(3)^2 - 4(1)(8) = 9 - 32 = -23$

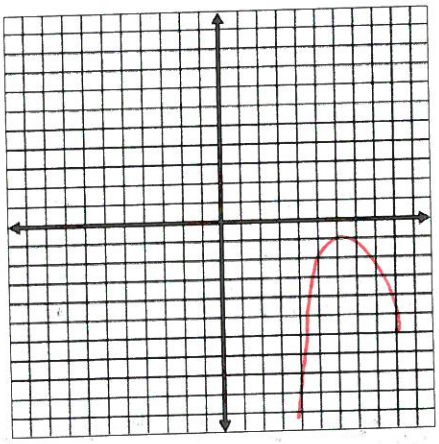
There are no real roots, but 2 complex roots instead.

7. Create a sketch of a quadratic function with a:

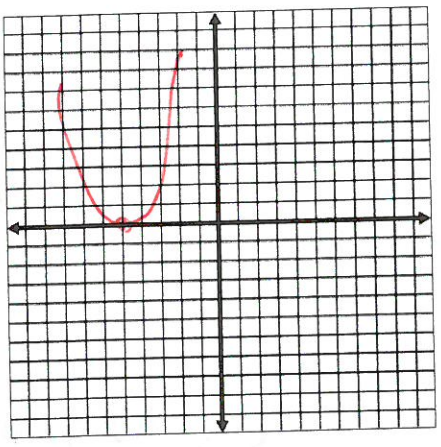
Discriminant >0



Discriminant <0



Discriminant =0



8. Simplify: $(6+i)-(-1+4i)$

$7-3i$

9. Simplify: $\frac{2+i}{3-5i}\left(\frac{3+5i}{3+5i}\right)=$

$\frac{6+10i+3i+5i^2}{9-25i^2}=\frac{1+13i}{34}$

10. Simplify: $(6+i)(-1+4i)$

$-6+24i-i+4i^2$

$-10+23i$

11. Simplify: $\sqrt{-72}$

$6i\sqrt{2}$

12. Simplify: i^{45}

i

13. If a toy rocket is launched vertically upward from ground level with an initial velocity of 180 feet per second, then its height h after t seconds is given by the equation $h(t) = -16t^2 + 180t$ (if air resistance is neglected).

a. How long will it take for the rocket to return to the ground?

$-16t^2 + 180t = 0$ $t = 0$ $t = \frac{45}{4} = 11.25$

$-4t(4t - 45) = 0$

It will take 11.25 sec to return.

b. After how many seconds will the rocket be 112 feet above the ground?

$-16t^2 + 180t = 112$ $0 = 4(4t^2 - 45t + 28)$ $t = 45 \pm \frac{\sqrt{(45)^2 - 4(4)(28)}}{8}$

$0 = 16t^2 - 180t + 112$

After 10.91 secs and .66 secs

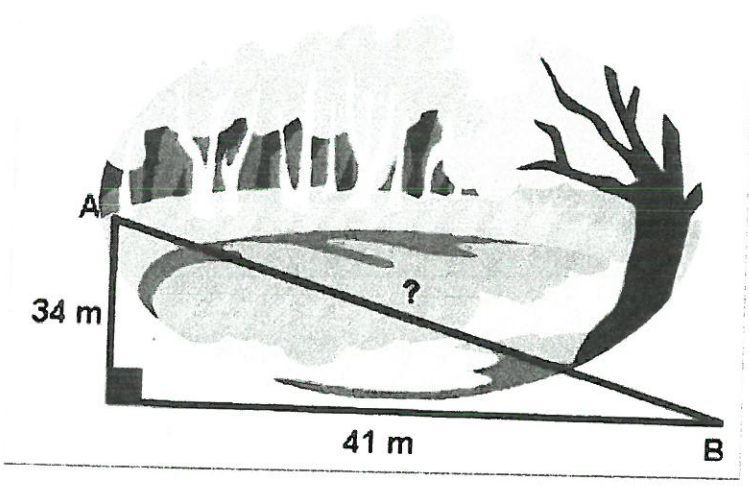
c. How long will it take the rocket to hit its maximum height?

The maximum height occurs at 5.63 seconds

d. What is the maximum height?

The maximum height is 506.25 ft

14.



To get from point A to point B you must avoid walking through a pond. To avoid the pond, you must walk 34 meters south and 41 meters east. To the *nearest meter*, how many meters would be saved if it were possible to walk through the pond?

$41^2 + 34^2 = c^2$ $(34 + 41) - 53 \approx$ 22m

$2537 = c^2$

$\approx 53.2_m = c$

53m

Polynomials

1) Find the zeros of the polynomial.

A. $f(x) = x^4 + 10x^2 + 25$ $(x^2+5)(x^2+5)$ $x = \pm i\sqrt{5}$

B. $f(x) = x^3 - 4x^2 - 25x$ $x(x^2-4x-25)$ $x = 0,\ x = 2 \pm \sqrt{29}$

$x = \frac{4 \pm \sqrt{4^2 - 4(1)(-25)}}{2}$ $x = \frac{4 \pm \sqrt{116}}{2}$ $\frac{4 \pm 2\sqrt{29}}{2} = 2 \pm \sqrt{29}$

2) Write a polynomial in standard form given zeros: 1, 3, -2

$y = (x-1)(x-3)(x+2) \rightarrow y = x^3 - 2x^2 - 5x + 6$

3) Divide the following:

A. $(x^3 + 12x^2 + 34x + 68) \div (x+9)$ $x^2 + 3x + 7 + \frac{5}{x+9}$

3A) -9 | 1 12 34 68 / -9 -27 -63 / 1 3 7 5

B. $(x^2 + 3x - 36) \div (x+2)$ $x + 1 + \frac{-38}{x+2}$

C. $(x^4 + 3x^2 + 1) \div (x^2 - 2x + 3)$ $x^2 + 2x + 4 + \frac{2x - 11}{x^2 - 2x + 3}$

3B) -2 | 1 3 -36 / -2 -2 / 1 1 -38

D. $(2x^4 + 4x^3 + 34x + 68) \div (x^2 + 2x - 3)$ $2x^2 + 6 + \frac{22x + 86}{x^2 + 2x - 3}$

4) How do you know if something is a factor of a polynomial? The remainder is zero when you plug into the function

5) Is $x+3$ a factor of $5x^3 + 18x^2 + 7x - 6$? Yes

-3 | 5 18 7 -6 / -15 -9 6 / 5 3 -2 0

6) Find $P(1)$ for $P(x) = 5x^3 + 18x^2 + 7x - 6$. $P(1) = 24$

1 | 5 18 7 -6 / 5 23 30 / 5 23 30 24

7) Sketch the polynomial $f(x) = -x^2(x+3)(x-4)$ without a calculator

Zeros: 0, -3, 4

Leading Coefficient(+/-): $-$

Degree: 4

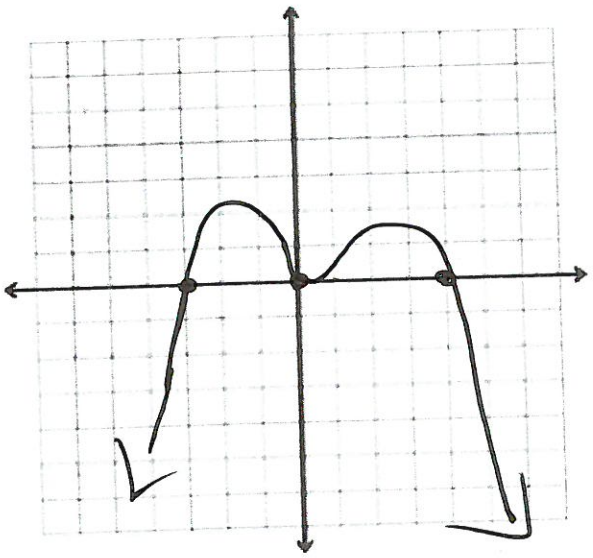
x-intercepts: 0, -3, 4

y-intercept: 0

Left end behavior: As $x \rightarrow -\infty$, $f(x) \rightarrow -\infty$

Right end behavior: As $x \rightarrow \infty$, $f(x) \rightarrow -\infty$

Behavior at the zeros:



3C) $x^2 - 2x + 3$ | $x^4 + 0x^3 + 3x^2 + 0x + 1$

$x^2 + 2x + 4$

$-x^4 + 2x^3 - 3x^2$ ↓

$2x^3 + 0x^2 + 0x$

$-2x^3 + 4x^2 - 6x$

$4x^2 - 6x + 1$

$-4x^2 + 8x - 12$

$2x - 11$

3D) $x^2 + 2x - 3$ | $2x^4 + 4x^3 + 0x^2 + 34x + 68$

$2x^2 + 0x + 6$

$-2x^4 - 4x^3 + 6x^2$ ↓ ↓

$0x^3 + 6x^2 + 34x + 68$

$-6x^2 - 12x + 18$

$22x + 86$

8) Graph with a calculator $(x^2+4)(x^2-1) = (x+2)(x-2)(x+1)(x-1)$

For the following function, $U(x) = x^4 - 5x^2 + 4$, find the following:

a. Number of expected zeros: 4

b. End Behavior:

As $x \to \infty$, $f(x) \to \infty$

As $x \to -\infty$, $f(x) \to \infty$

c. Domain: $\mathbb{R}$

d. Range: $[-2.25, \infty)$

e. Absolute Minimum: $(\pm 1.58, -2.25)$

f. Absolute Maximum: None

g. Relative Maximum: $(0, 4)$

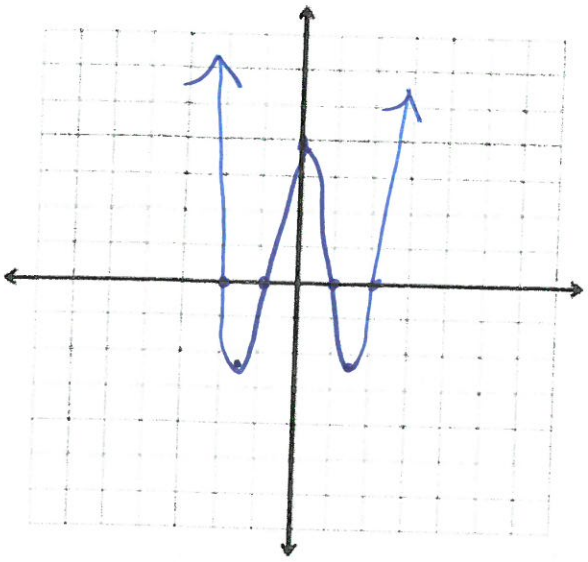
h. Relative Minimum: $(\pm 1.58, -2.25)$

i. Y-Intercept: $(0, 4)$

j. Zeros: $(-2,0)(-1,0)(1,0)(2,0)$

k. Increasing: $(-1.58, 0)\ (1.58, \infty)$

l. Decreasing: $(-\infty, -1.58)\ (0, 1.58)$



9) Expand $(s-2v)^5$ GT ONLY $1(s)^5 + 5(s)^4(-2v) + 10(s)^3(-2v)^2 + 10(s)^2(-2v)^3 + 5(s)^1(-2v)^4 + 1(-2v)^5$

$s^5 - 10s^4v + 40s^3v^2 - 80s^2v^3 + 80sv^4 - 32v^5$

10) Expand $(x-5)^6$ GT ONLY $1(x)^6 + 6(x)^5(-5)^1 + 15(x)^4(-5)^2 + 20(x)^3(-5)^3 + 15(x)^2(-5)^4 + 6(x)^1(-5)^5 + 1(-5)^6$

$x^6 - 30x^5 + 375x^4 - 2500x^3 + 9375x^2 - 18750x + 15625$

Radicals

Simplify

1) $\sqrt[3]{24x^5y^{12}z}$ $2xy^4\sqrt[3]{3x^2z}$

2) $12\sqrt{160} + 8\sqrt{363} - 2\sqrt{125}$ $48\sqrt{10} + 88\sqrt{3} - 10\sqrt{5}$

3) $\sqrt{300a^3b^8}$ $= 10ab^4\sqrt{3a}$

4) $\sqrt[4]{81x^{12}y^{36}}$ $3x^3y^9\sqrt{3}$

5) $(-8)^{2/3} = 4$

6) $(81x^4y^{-2}z^{-5})^{1/3} = \frac{\sqrt[3]{81x^4}}{\sqrt[3]{y^2z^5}} = \frac{3x}{z}\sqrt[3]{\frac{3x}{y^2z^2}}$

7) $\sqrt{20x^4y^7} \cdot \sqrt{15x^4y^2}$ $10x^4y^4\sqrt{3y}$

8) $(3+\sqrt{18})(2-\sqrt{3})$ $(3+3\sqrt{2})(2-\sqrt{3})$ $6 - 3\sqrt{3} + 6\sqrt{2} - 3\sqrt{6}$

9) $\frac{12y^2\sqrt{y^2z^5}}{\sqrt[4]{5x^2}} = \frac{12y^4\sqrt{125x^2}}{5x}$

10) $\frac{\sqrt{14}+12}{\sqrt{3}+\sqrt{2}}\left(\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}\right)$ $\frac{\sqrt{42}-\sqrt{28}+12\sqrt{3}-12\sqrt{2}}{3-2} =$ $\sqrt{42} + 2\sqrt{7} + 12\sqrt{3} + 12\sqrt{2}$

11) $\sqrt{18} - 3\sqrt{108} + \sqrt{24}$ $3\sqrt{2} - 3\cdot 6\sqrt{3} + 2\sqrt{6} =$ $3\sqrt{2} - 18\sqrt{3} + 2\sqrt{6}$

12) $5\sqrt[3]{64x} - 7\sqrt[3]{64x}$ $20\sqrt[3]{x} - 28\sqrt[3]{x}$ $-8\sqrt[3]{x}$

Solve for x.

13) $\sqrt{x+2}=8$ $x=62$

14) $=\sqrt{x+13}+7$ $x=3,12$

15) $\sqrt[4]{x-2}+6=0$ No solution

16) $\sqrt{12-3x}=\sqrt{x-3}$ $x=15/4$

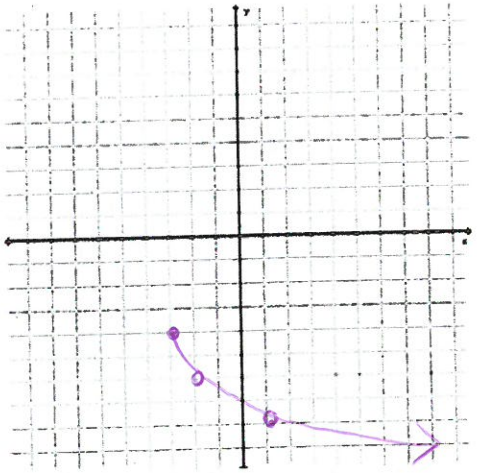
Find the following and graph. Plot 3 accurate points.

17) $F(x)=-2\sqrt{x+3}-4$

Domain: $x \geq -3$

Range: $(-\infty, -4]$

Shifts: left 3 down 4, stretch by 2, reflect over x-axis

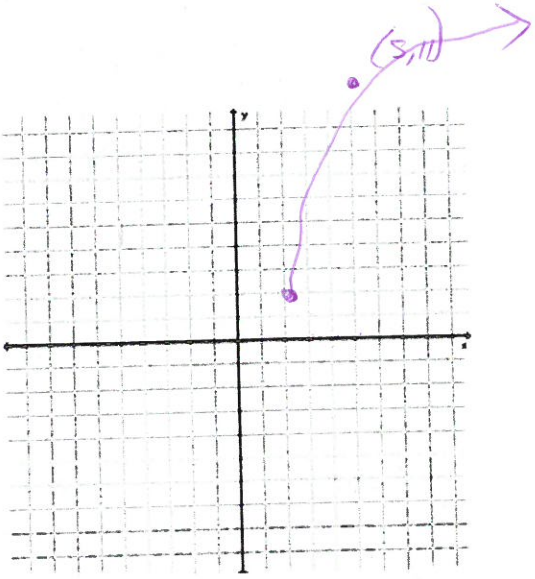


18) $U(x)=3\sqrt{3x-6}+2$

Domain: $x \geq 2$

Range: $y \geq 2$

Shifts: right 2, up 2, vertical stretch by 3, horizontal stretch by 3



19) Find the Domain AND Range of the function $y=\sqrt{x^2-6x-7}$ (GT only)

$x^2-6x-7=0$ $(x-7)(x+1)=0$ D: $x \leq -1 \cup x \geq 7$ R: $y \geq 0$

20) Find the Domain AND Range of the function $W(x)=\sqrt{x^2+16}$ (GT only)

D: $\mathbb{R}$ R: $y \geq 4$

Rationals

Find the following and graph. Include 3 accurate points for each section of the graph.

1) $D(x) = \dfrac{3x^3+9x^2}{x^2-x-12} = \dfrac{3x^2(x+3)}{(x+3)(x-4)}$

Domain: $(-\infty,-3)\cup(-3,4)\cup(4,\infty)$

Holes: $x=-3$

V. A: $x=4$

H. A: No HA

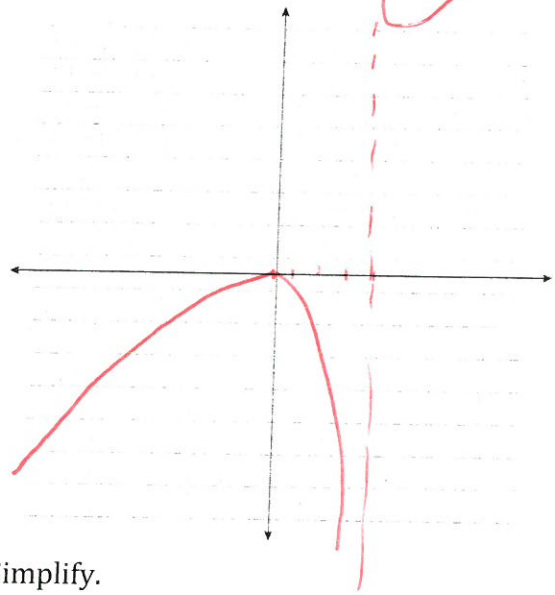
2) $U(x) = \dfrac{3x^2-x}{x^2+4}$ $\dfrac{x(3x-1)}{x^2+4}$

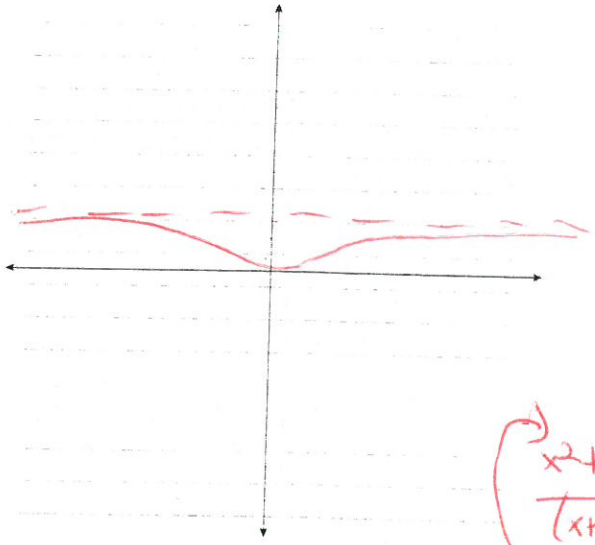
Domain: $\mathbb{R}$

Holes: None

V. A: None

H. A: $y=3$





Simplify.

3) $\dfrac{3x^2-12}{x^2+5x+6} = \dfrac{3(x-2)}{x+3}$ $\dfrac{3(x-2)(x+2)}{(x+2)(x+3)} =$

4) $\dfrac{x+4}{x-3} - 4\left(\dfrac{x-3}{x-3}\right)$ $\dfrac{-3x+16}{x-3}$

5) $\dfrac{x+4}{x^2+7x+12} - \dfrac{x}{x^2-16}$ $\dfrac{x+4}{(x+3)(x+4)} \quad \dfrac{x}{(x+4)(x-4)}$ $\dfrac{x^2+16-x(x+3)}{(x+3)(x+4)(x-4)}$ $\dfrac{-3x+16}{(x+3)(x+4)(x-4)}$

6) $\dfrac{x^2-9}{x^2-5x+6} * \dfrac{x-3}{x^2+8x+15}$

$\dfrac{(x+3)(x-3)}{(x-6)(x+1)} \cdot \dfrac{(x-3)}{(x+3)(x+5)} = \dfrac{(x-3)^2}{(x-6)(x+1)(x+5)}$

7) $\dfrac{x^2-5x-14}{x^2-7x-8} \div \dfrac{2x^2-98}{x^2+10x+16}$ $\dfrac{(x-7)(x+2)}{(x-8)(x+1)} \cdot \dfrac{(x+8)(x+2)}{2(x+7)(x-7)}$

8) $\dfrac{x}{x+4} \cdot \dfrac{\dfrac{x^2+6x+8}{x+10}}{\dfrac{x^2}{x+20}}$

$\dfrac{x}{x+4} \cdot \dfrac{x^2+6x+8}{x+10} \cdot \dfrac{x+20}{x^2} = \dfrac{x}{x+4} \cdot \dfrac{(x+4)(x+2)}{x+10} \cdot \dfrac{x+20}{x} = \dfrac{(x+2)(x+20)}{x(x+10)}$

9) $\dfrac{\dfrac{1}{x}-\dfrac{3}{x-2}}{\dfrac{2}{x}+\dfrac{5}{x-2}}$ $LCD = x(x-2)$ $\dfrac{(x-2)-3(x)}{2(x-2)+5(x)} = \dfrac{x-2-3x}{2x-4+5x} = \dfrac{-2x-2}{7x-4}$ or $\dfrac{-2(x+1)}{7x-4}$

$\dfrac{(x+2)^2(x+8)}{2(x-8)(x+1)(x+7)}$

10) $\left(\frac{4}{3}-\frac{4}{x}=\frac{16}{3x}\right)3x$

$4x-12=16$

$4x=28$

$x=7$

11) $(x-2)(x-6)=(x-3)-(2x-7)(x+1)$

$x^2-8x+12=x-3-(2x^2-5x-7)$

$x^2-8x+12=-2x^2+6x+4$

$3x^2+14x+8=0$

$(3x-2)(x+4)=0$

LCD $=(x-6)(x+1)$

$x=\frac{2}{3}, 4$

Solve for x.

10) $\frac{4}{3}-\frac{4}{x}=\frac{16}{3x}$

LCD $=3x$ $\quad x=7$

11) $\frac{x-2}{x+1}=\frac{x-3}{x^2-5x-6}-\frac{2x-7}{x-6}$

$(x-6)(x+1)$

12) $\frac{2}{x+3}-\frac{3}{4-x}=\frac{2x+2}{x^2-x-12}$

$-1(x-4)$

$(x+3)(x-4)\left(\frac{2}{x+3}+\frac{3}{x-4}=\frac{2x+2}{(x-4)(x+3)}\right)$

LCD $=(x+3)(x-4)$

$2(x-4)+3(x+3)=2x+2$

$2x-8+3x+9=2x+2$

$5x+1=2x+2$

$3x=1$

$x=\frac{1}{3}$

Trigonometry

1) Sketch the arc for $\frac{5\pi}{3}$.

2) Sketch the arc for $-\frac{\pi}{6}$ and find the sine, cosine and tangent of the value.

$\sin\left(\frac{-\pi}{6}\right)=\frac{-1}{2}$

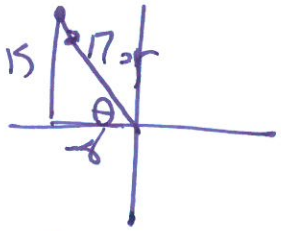
$\cos\left(\frac{-\pi}{6}\right)=\frac{\sqrt{3}}{2}$

$\tan\left(\frac{-\pi}{6}\right)=\frac{-\sqrt{3}}{3}$

3) If cos<0and sin<0, what quadrant is in? III

4) What is the $\cos(\frac{5\pi}{6})$? IV $\quad -\frac{\sqrt{3}}{2}$

5) Sketch the triangle formed by the coordinates (-8, 15) and find the six trig functions for θ.



$(-8)^2+15^2=r^2$

$17=r$

$\sin\theta=\frac{15}{17}$

$\cos\theta=\frac{-8}{17}$

$\tan\theta=-\frac{15}{8}$