Name _____ Trigonometry

Date _____ Review for first part of Identities Unit

Find the General Solution.

1. $\sin\theta = \dfrac{-\sqrt{3}}{2}$

$\theta = \frac{4\pi}{3} + 2\pi k$

$\theta = \frac{5\pi}{3} + 2\pi k$

2. $\sec^2\theta = 4$

$\sec\theta = \pm 2$

$\cos\theta = \pm \frac{1}{2}$

$\theta = \frac{\pi}{3} + 2\pi k, \frac{2\pi}{3} + 2\pi k, \frac{4\pi}{3} + 2\pi k, \frac{5\pi}{3} + 2\pi k$

3. $\tan x + 1 = 1$

$-1 \quad -1$

$\tan x = 0$

$x = 0, \pi$

$x = 0 + \pi k$ or just πk

Find the solution on the interval $0 \le \theta < 2\pi$.

4. $1 - 2\cos\theta = 0$

$-2\cos\theta = -1$

$\cos\theta = \frac{1}{2}$

$\theta = \frac{\pi}{3} + 2\pi k$

$\theta = \frac{5\pi}{3} + 2\pi k$

5. $\tan x + \sqrt{3} = 0$

$\tan x = -\sqrt{3}$

$x = \frac{2\pi}{3}, \frac{5\pi}{3}$

6. $\csc^2\theta - 4 = 0$

$\csc^2\theta = 4$

$\csc\theta = \pm 2$

$\sin\theta = \pm \frac{1}{2}$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

7. $4\sin^2 x = 1 + 4\cos x$

$4(1 - \cos^2 x) = 1 + 4\cos x$

$0 = 4\cos^2 x + 4\cos x - 3$

$0 = (2\cos x - 1)(2\cos x + 3)$

$\cos x = \frac{1}{2}$ $\quad$ $\cos x = -\frac{3}{2}$ no solution!

$x = \frac{\pi}{3}, \frac{5\pi}{3}$

8. $(\tan\theta - 1)(\sec\theta - 1) = 0$

$\tan\theta = 1$ $\quad$ $\sec\theta = 1$

$\theta = \frac{\pi}{4}, \frac{5\pi}{4}$ $\quad$ $\cos\theta = \frac{1}{1}$

$\theta = 0$

$\theta = 0, \frac{\pi}{4}, \frac{5\pi}{4}$

9. $\cos(2x) = 1$

$2x = 0 + 2\pi k$

$x = 0 + \pi k$

$x = 0, \pi$

10. $\sin^2\theta - \cos^2\theta = 0$

$(\sin\theta - \cos\theta)(\sin\theta + \cos\theta) = 0$

$\sin\theta = \cos\theta$ $\quad$ $\sin\theta = -\cos\theta$

Think about it! Where is the $\sin\theta = \cos\theta$?

$\theta = \frac{\pi}{4}, \frac{5\pi}{4}$ $\quad$ $\theta = \frac{3\pi}{4}, \frac{7\pi}{4}$

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Simplify to **one** term.

11. $\csc\theta\tan\theta$

$\frac{1}{\sin\theta}\cdot\frac{\sin\theta}{\cos\theta}$

$\frac{1}{\cos\theta} = \sec\theta$

12. $\frac{\sec\theta}{\tan\theta}$

$= \frac{\frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}} = \frac{1}{\sin\theta}$

$= \csc\theta$

13. $\cos^2\theta\left(1+\tan^2\theta\right)$

$\cos^2\theta(\sec^2\theta)$

$\frac{\cos^2\theta}{1}\cdot\frac{1}{\cos^2\theta} = 1$

14. $\sin\theta\csc\theta - \cos^2\theta$

$\frac{\sin\theta}{1}\cdot\frac{1}{\sin\theta} - \cos^2\theta$

$1-\cos^2\theta = \sin^2\theta$

15. $\frac{\cot^4\theta+\cot^2\theta}{\csc^2\theta}$

$= \frac{\cot^2\theta(\cot^2\theta+1)}{\csc^2\theta}$

$= \cot^2\theta\left(\frac{\csc^2\theta}{\csc^2\theta}\right) = \cot^2\theta$

16. $\cos\theta(\tan\theta+\cot\theta)$

$= \cos\theta\left(\frac{\cos\theta}{\sin\theta}+\frac{\sin\theta}{\cos\theta}\right)$

$= \frac{\cos^2\theta}{\sin\theta} + \sin\theta\left(\frac{\sin\theta}{\sin\theta}\right)$

$= \frac{\cos^2\theta}{\sin\theta} + \frac{\sin^2\theta}{\sin\theta} = \frac{\cos^2+\sin^2\theta}{\sin\theta}$

$= \frac{1}{\sin\theta}$

$= \csc\theta$

Prove the following identities

17. $\frac{1+\tan\theta}{1-\tan\theta} = \frac{1+\cot\theta}{-1+\cot\theta}$

$= \frac{1+\frac{1}{\tan\theta}}{-1+\frac{1}{\tan\theta}}$ ← Get a common Denominator → $\frac{1+\sin\theta}{1+\sin\theta} - \frac{\cos^2\theta}{1+\sin\theta} = \sin\theta$

$= \frac{\tan\theta+1}{-\tan\theta+1}$

$\frac{1+\tan\theta}{1-\tan\theta} = \frac{1+\tan\theta}{1-\tan\theta}$ ✓

$= \frac{1+\sin\theta-(1-\sin^2\theta)}{1+\sin\theta} = \sin\theta$

$= \frac{1+\sin\theta-1+\sin^2\theta}{1+\sin\theta} = \sin\theta$

$= \frac{\sin\theta+\sin^2\theta}{1+\sin\theta} = \sin\theta$

$= \frac{\sin\theta(1+\sin\theta)}{(1+\sin\theta)} = \sin\theta$ ✓

18. $1-\frac{\cos^2\theta}{1+\sin\theta} = \sin\theta$

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Prove the following identities

19. $\dfrac{\sin x}{1+\cos x}+\dfrac{1+\cos x}{\sin x}=2\csc x$

$\dfrac{\sin^2 x+1+\cos^2 x+2\cos x}{\sin x(1+\cos x)}=2\csc x$

$\dfrac{2+2\cos x}{\sin(1+\cos x)}=2\csc x$

$2\cdot\dfrac{1}{\sin x}\cdot\dfrac{2(1+\cos x)}{\sin x(1+\cos x)}=2\csc x$ ✓

$2\cdot\csc x$

20. $\tan\theta\cot\theta-\sin^2\theta=\cos^2\theta$

$\dfrac{\tan\theta}{1}\cdot\dfrac{1}{\tan\theta}-\sin^2\theta=\cos^2\theta$

$1-\sin^2\theta=\cos^2\theta$

$\cos^2\theta=\cos^2\theta$ ✓

21. $\dfrac{\sec\theta}{\csc\theta}+\dfrac{\sin\theta}{\cos\theta}=2\tan\theta$

$\dfrac{\frac{1}{\cos\theta}}{\frac{1}{\sin\theta}}+\dfrac{\sin\theta}{\cos\theta}=2\tan\theta$

$\dfrac{\sin\theta}{\cos\theta}+\dfrac{\sin\theta}{\cos\theta}=2\tan\theta$

$\tan\theta+\tan\theta=2\tan\theta$

$2\tan\theta=2\tan\theta$ ✓

22. $\dfrac{\csc\theta-1}{\cot\theta}=\dfrac{\cot\theta}{\csc\theta+1}$

$\dfrac{\csc\theta-1}{\cot\theta}\left(\dfrac{\cot\theta}{\cot\theta}\right)=\dfrac{\cot\theta}{\csc\theta+1}$

$\dfrac{\cot\theta(\csc\theta-1)}{\cot\theta}=\dfrac{\cot\theta}{\csc\theta+1}$

$\dfrac{\cot\theta(\csc\theta-1)}{\csc^2-1}=\dfrac{\cot\theta}{\csc\theta+1}$

$\dfrac{\cot\theta(\csc\theta-1)}{(\csc\theta-1)(\csc\theta+1)}$

$\dfrac{\cot\theta}{\csc\theta+1}=\dfrac{\cot\theta}{\csc\theta+1}$ ✓