Name Key Algebra 2GT

Practice: Sequences, Series, Exponential Growth &Decay, Compounding

1. Find the arithmetic mean of the 5th and 7th terms in the sequence $a_n = 4 - \frac{3}{2}n$.

$a_5 = 4 - \frac{3}{2}(5) = 4 - 7.5 = -3.5$ $a_7 = 4 - \frac{3}{2}(7) = 4 - 10.5 = -6.5$ $\frac{-3.5 + -6.5}{2} = \boxed{-5}$

2. Find the geometric mean of the 6th and 8th terms in the sequence $a_n = 2\left(\frac{1}{3}\right)^{(n-1)}$

$a_6 = 2\left(\frac{1}{3}\right)^5 = \frac{2}{3^5}$ $a_8 = 2\left(\frac{1}{3}\right)^7 = \frac{2}{3^7}$ mean $\sqrt{\frac{2}{3^5} \cdot \frac{2}{3^7}} = \sqrt{\frac{2^2}{3^{12}}} = \frac{2}{3^6} = \boxed{\frac{2}{729}}$

3. Find the missing terms in the geometric sequence $5, -2, \frac{4}{5}, \underline{-8/125}, \underline{16/125}, -\frac{32}{625}$

$-2 = 5(r)^{(2-1)}$ $A_n = 5\left(-\frac{2}{5}\right)^{n-1}$

$-2 = 5 \cdot r$ $r = -\frac{2}{5}$ $\boxed{A_4 = -8/125}$ $\boxed{A_5 = 16/125}$

4. Find the sum of the first 12 terms of $8 + 4 + 2 + 1 + \ldots$

$a_1 = 8$ $S_{12} = 8\left(\frac{1 - (1/2)^{12}}{1 - \frac{1}{2}}\right) = 16\left(1 - \frac{1}{2^{12}}\right) \approx \boxed{16}$

$r = 1/2$

For #5-6, determine if you can find the sum of each infinite series. If so, find it. If not explain why not.

5. $3 - \frac{9}{2} + \frac{27}{4} - \frac{81}{8} + \ldots$ no, this series diverges

$r = -3/2$ $|-3/2| > 1$

6. $3 - \frac{3}{4} + \frac{3}{16} - \frac{3}{64} + \ldots$ yes, this series converges $S = \frac{a_1}{1-r} = \frac{3}{1-(-1/4)} = \frac{3}{5/4}$

$r = -1/4$ $|-1/4| < 1$ $12/5 = \boxed{2.4}$

7. An arithmetic sequence has a 2[nd] term of $x+1$ and a 6[th] term of $x+17$.

a_2 a_6

a. Find the explicit formula for this sequence.

$d = \frac{x + 17 - (x+1)}{6-2} = \frac{16}{4} = 4$ $x + 1 = a_1 + 4(2-1)$

$x - 3 = a_1$

$a_n = x - 3 + 4(n-1)$

b. Determine the 20[th] term.

$a_{20} = x - 3 + 4(20-1) = x - 3 + 76 = \boxed{x + 73 = a_{20}}$

8. An infinite geometric series with a first term of 12 converges to 96. What is the common ratio?

$a_1 = 12$ $S = \frac{a_1}{1-r}$ $96(1-r) = 12$

$S = 96$ $1 - r = 1/8$

$96 = \frac{12}{1-r}$

$\boxed{r = 7/8}$

Name________________________________ Algebra 2GT

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9. Using the formulas for sequences and series, find the sum of $64+96+144+...+729$.

$a_1 = 64$ $a_n = 64(3/2)^{n-1}$ $S_7 = 64\left(\frac{1-(3/2)^7}{1-(3/2)}\right)$

$r = 3/2$ $729 = 64\left(\frac{3}{2}\right)^{n-1}$

$n = 7$ $\boxed{S_7 = 2059}$

10. The population of a certain animal species decreases at a rate of 3.5% per year. You have counted 80 of the animals in the habitat you are studying.

a. Predict the number of animals remaining in the habitat in 5 years.

$A = 80(1-0.035)^5 = 66.95$ $\boxed{\text{About 67 animals}}$

b. Estimate the number of years until the population first drops below 15 animals.

$15 = 80(0.965)^x$ In about 47 years

$x = 46.99$

11. In 1995 there were about 34 million people using cellular phones. Cell phone usage grew about 22% each year from 1995 to 2003.

a. Estimate the number of cell phone users in 2000. $2000 - 1995 = 5$

$34(1.22)^5 = \boxed{\text{91.89 million cell phone users}}$

b. Estimate the year at which the number of cell phone users will reach 2 billion.

$2,000 = 34(1.22)^x$ In about 20 years from 1995...

$x = 20.49$ $\boxed{\text{In 2015}}$

12. A scientist has discovered a new strain of bacteria that doubles every half hour. The bacteria culture contained 1000 bacteria at 10:00 am.

a. Predict the number of bacteria at 2:30 pm. 2:30 is 4 1/2 hours after 10am therefore there are 9 half hours... $x = 9$

$A = 1000(2)^x$

$A = 1000(2)^9 = \boxed{\text{512,000 bacteria}}$

b. Estimate the time that the bacteria will surpass 1,000,000 bacteria.

$1,000,000 = 1,000(2)^x$ In about 10 half hours,

$x = 9.97$ So about $\boxed{\text{3:00 pm}}$

Name_______________________________ Algebra 2GT

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13. The first credit card that you got charges 12.49% interest annually to its customers and compounds that interest monthly. Within one day of getting your first credit card, you max out the credit limit by spending $1,200.00. If you do not buy anything else on the card and you do not make any payments, how much money would you owe the company after 6 months?

$A = 1{,}200\left(1+\frac{.1249}{12}\right)^{(12\cdot\frac{1}{2})}$

$A = \$1{,}276.92$

14. You win the lottery and get $1,000,000. You decide that you want to invest all of the money in a savings account. However, your bank has two different plans. In 5 years from now, which plan will provide you with more money?

Plan 1

The bank gives you a 6% interest rate and compounds the interest each month.

Plan 2

The bank gives you a 12% interest rate and compounds the interest every 2 months.

Plan 1: $1{,}000{,}000\left(1+\frac{.06}{12}\right)^{(12.5)} = \$1{,}348{,}850.15$

Plan 2: $1{,}000{,}000\left(1+\frac{.12}{2}\right)^{(2.5)} = \$1{,}790{,}847.70$

I will choose $\boxed{\text{Plan 2}}$ because it gives me more money!

15. An account earning % interest compounded continuously for 10 years would have a balance of how much if the principal was $5500.00

$A = 5500e^{(.025\cdot 10)}$

$A = \$7{,}062.14$

16. What was the principal for a continuously compounded account earning 3.9% for 15 years if it currently has a balance of $2,500,000.00?

$2{,}500{,}000 = Pe^{(.039\cdot 15)}$

$2{,}500{,}000 = P(1.79)$ → If you rounded to 1.79 here instead of using what was on the calculator,

$\$1{,}392{,}764.66 = P$

$P = \$1{,}396{,}648.05$