Name ________________________________ **AP Stats Review for Chapter 18 Quiz**

Know this table:

	Assumptions	Mean of Sampling Dist	St. Dev of Sampling Dist
Proportions $\hat{p}$	a. SRS. b. Population is more than 10 times the sample size. c. $np \geq 10$ *and* $n(1-p) \geq 10$	p	$\sqrt{\frac{p(1-p)}{n}}$
Means $\bar{x}$	a. SRS b. Population is normal OR Sample size is large ($n \geq 30$)	μ	$\frac{\sigma}{\sqrt{n}}$

1. An opinion poll asks, "Are you afraid to go outside at night within a mile of your home because of crime?" Suppose that the proportion of all adults who would say "Yes" to this question is p = 0.4. An SRS of size n = 200 is taken.

a. In the space below, calculate np and n (1 - p). Show your work. What implications do these results have for our sketch of the sampling distribution of $\hat{p}$?

$np = 200(.4) = 80$
$n(1-p) = 200(.6) = 120 \geq 10$ ✓

Since both np and n(1-p) are greater than or equal to 10, we can use the Normal model to approximate the Probability

b. Is $\hat{p}$ an unbiased estimator of p? How do you know? What implications does the answer to this question have for our sketch of the sampling distribution of $\hat{p}$?

Yes, since we have an SRS, $\hat{p}$ is an unbiased estimator of p. So, this means that the center of our sampling distribution will be p=.4

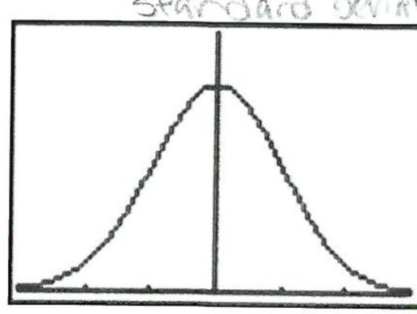
c. In the space below, show the calculation of the standard deviation of the sampling distribution of $\hat{p}$? Under what conditions can this formula for the standard deviation of $\hat{p}$ be reasonably accurate? What implications does the answer to your calculation have for our sketch of the sampling distribution of $\hat{p}$?

$\sigma_{\hat{p}} = \sqrt{\frac{(.4)(.6)}{200}} = .0346$

This standard deviation is reasonably accurate when the sample size is less than 10% of the population.

This means that our sampling distribution has a standard deviation of $\sigma_{\hat{p}} = .0346$ (More narrow than the population)

d. Sketch and label the sampling distribution of the sample proportion at right. How can we cut the spread of this distribution in half?



p=.4
σ=.0346

If we increase the sample size by a factor of 4, the spread is cut in half. (In this case if the sample size increases to 800.

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2. The average outstanding bill for delinquent customer accounts for a national department store chain is \$187.50 with a standard deviation of \$54.50. A simple random sample of 50 delinquent accounts is taken.

$\mu = 187.50 \quad \sigma = 54.50 \quad n = 50$

a. Is the sampling distribution of $\bar{x}$ approximately normal? Justify your assertion in the space below. What implications does the answer to this question have for our sketch of the sampling distribution of $\bar{x}$?

Yes, since the sample size (n=50) is greater than or equal to 30, the sampling distribution is approximately Normal. This means our sampling distribution can be approximated by the Normal model.

b. Is $\bar{x}$ an unbiased estimator of μ? Justify your assertion. What implications does the answer to this question have for our sketch of the sampling distribution of $\bar{x}$?

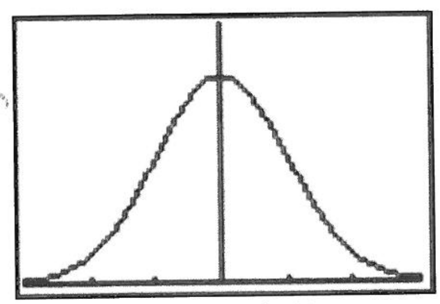
Yes, since we have an SRS, $\bar{x}$ is an unbiased estimator of μ. This means that the center of our sampling distribution will be $\mu = 187.50$

c. Can we use the formula for the standard deviation of $\bar{x}$? Justify your assertion. Calculate the standard deviation of $\bar{x}$. What implications does the answer to this question have for our sketch of the sampling distribution of $\bar{x}$?

Yes, since our sample size (n=50) is less than 10% of our population, we can use the formula for $\sigma_{\bar{x}}$. This means that our sampling distribution has a standard deviation of

$\sigma_{\bar{x}} = \frac{54.50}{\sqrt{50}} = 7.7075$ which is more narrow than the population

d. Sketch and label the sampling distribution of the sample mean at right. How can we cut the spread of this distribution to 1/4 th of its present size?



$\mu = 187.50$

$\sigma_{\bar{x}} = 7.7075$

If we multiply the sample size by 16, we will cut the spread of the distribution by $\frac{1}{4}$. In this case our sample size would go from 50 to 800.

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3. The Central Limit Theorem has basically three parts to it. Start with a population that has ANY shape. Let this population have mean μ and standard deviation σ. Pick n sufficiently large (at least 30) and take all samples of size n. Compute the mean of each of these samples. Then

a. The sampling distribution is approximately Normal

b. The center of the sampling distribution is μ

c. The standard deviation $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

Solve the following problems

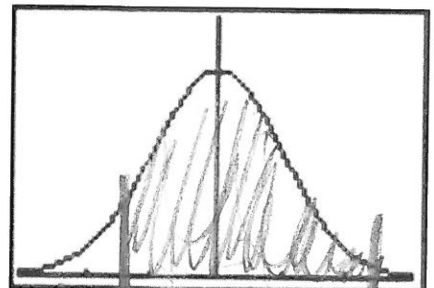
4. The strength of paper coming from a manufacturing plant is known to be 25 pounds per square inch with a standard deviation of 2.3. In a simple random sample of 40 pieces of paper, what is the probability that the mean strength is between 24.5 and 26.2 pounds per square inch?

$\mu = 25$ $\sigma = 2.3$ $n = 40$

a. Verify that the assumptions necessary to answer this question are satisfied. Then draw and label the sampling distribution at the right. Shade the region that an

- SRS is indicated ✓
- 40 < 10% of all pieces of paper ✓
- 40 ≥ 30 ✓

$\sigma_{\bar{x}} = \frac{2.3}{\sqrt{40}} = .3637$



b. Calculate the answer to the question in the space below, using Table A only. Do not use normalcdf!

$P(24.5 \le \bar{x} \le 26.2) = P(\bar{x} \le 26.2) - P(\bar{x} \le 24.5) =$

$z = \frac{24.5 - 25}{.3637} = -1.3748$ $P(z \le 3.3) - P(z \le -1.37) =$

$z = \frac{26.2 - 25}{.3637} = 3.299$ $.9995 - .0853 = \boxed{.9142}$

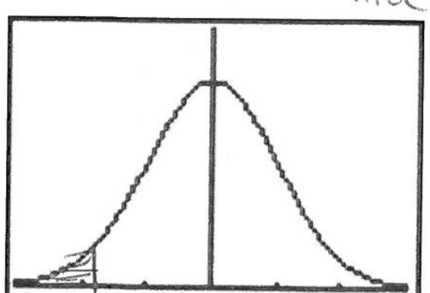
P-Value

c. In a simple random sample of 40 pieces of paper, what is the probability that the mean strength is less than 23.4 pounds? Use your calculator to do this, writing all necessary information to allow the reader to follow along. Sketch and label at right.

$P(\bar{x} \le 23.4)$

Normcdf(-1E99, 23.4, 25, .3637) = .00000...

P-Value



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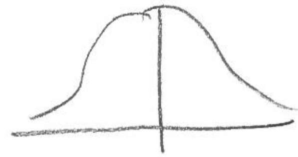
5. A sample of size 49 is drawn from a normal population with a mean of 63 and a standard deviation of 14 What are the mean and standard deviation of the distribution of sample means? Sketch this distribution in the space below. $n = 49$ Normal $\mu = 63$, $\sigma = 14$

- SRS is assumed
- Population is Normal (CLT)

$\sigma_{\bar{x}} = \frac{14}{\sqrt{49}} = 2$



$\mu = 63$
$\sigma_{\bar{x}} = 2$

6. A sample of size 25 is drawn from a normal population with a mean of 62. If the standard deviation of the distribution of sample means is 3.5, what is the standard deviation of the original population?

$n = 25$ Normal $\mu = 62$ $\sigma_{\bar{x}} = 3.5 = \frac{\sigma}{\sqrt{n}}$

$3.5 = \frac{\sigma}{\sqrt{25}}$

$\boxed{17.5 = \sigma}$

7. The distribution of SAT Math scores of students taking Calculus I at a large university is skewed left with a mean of 625 and a standard deviation of 44.5. If random samples of 100 students are repeatedly taken, describe the sampling distribution of the sample means, giving the shape, center, and spread.

$\mu = 625$ $\sigma = 44.5$

- SRS is indicated
- 100 < 10% of population
- Sample size $(n = 100) \geq 30$

We can apply CLT

$\mu = 625$
$\sigma_{\bar{x}} = 4.45$

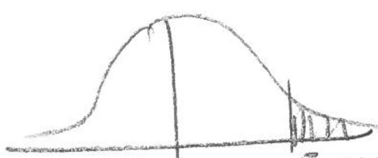
$\sigma_{\bar{x}} = \frac{44.5}{\sqrt{100}} = 4.45$

The sampling distribution is Normal with a center at $\mu = 625$ and a spread of $\sigma_{\bar{x}} = 4.45$

8. At a large bank, account balances are normally distributed with a mean of \$1,637.52 and a standard deviation of \$632.16. What is the probability that a simple random sample of 400 accounts has a mean that exceeds \$1,650? Be sure to verify assumptions.

Normal $\mu = 1637.52$ $\sigma = 632.16$ $n = 400$ $\bar{x} = 1650$

- SRS is indicated
- 400 < 10% of all accounts
- Population is Normal!



$\bar{x} = 1650$
$\mu = 1637.52$
$\sigma_{\bar{x}} = 31.608$

$\sigma_{\bar{x}} = \frac{632.16}{\sqrt{400}} = 31.608$

$z = \frac{1650 - 1637.52}{31.608} = .3948$

$P(\bar{x} > 1650) = 1 - P(\bar{x} \leq 1650) =$
$= 1 - P(z \leq .39) = 1 - .6517 = \boxed{.3483}$

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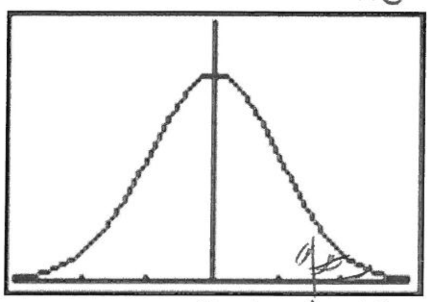
9. In a large urban school district, it has been accepted that the percentages of boys and girls having a 3.0 average or higher are both approximately 10%. A random sample of 110 girls indicated that 15% of the girls fit in this category, while a second independent random sample of 120 boys indicated that 12% of the boys performed at this level. Sketch and label the sampling distributions of the sample proportions for both the boys and girls.

$P_G = .1$ $\quad$ $P_B = .1$

$\hat{P}_G = .15$ $\quad$ $\hat{P}_B = .12$

$n = 110$ $\quad$ $n = 120$

Girls



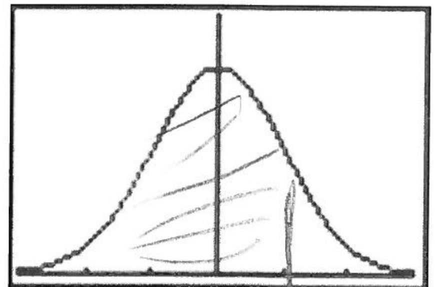
$p = .1 \quad \hat{p} = .15$

$\sigma_{\hat{p}} = .0286$

- SRS is indicated ✓
- $110 < 10\%$ of all girls ✓
- $110(.1) = 11$ ✓
  $110(.9) = 99$

$\sigma_{\hat{p}} = \sqrt{\frac{(.1)(.9)}{110}} = .0286$

Boys



$p = .1 \quad \hat{p} = .12$

$\sigma_{\hat{p}} = .0154$

- SRS is indicated
- $120 <$ 10% of all boys
- $120(.1) = 12$ ✓
  $120(.9) = 108$

$\sigma_{\hat{p}} = \sqrt{\frac{(.1)(.9)}{120}} = .0274$

10. What is the probability that a SRS of n = 110 girls will show that at least 15% have a 3.0 average? Show all work.

$P(\hat{p} \geq .15) = 1 - P(\hat{p} \leq .15) =$

$Z = \frac{.15 - .1}{.0286} = 1.7483$

$1 - P(Z \leq 1.75) =$

$1 - .9599 = \boxed{.0401}$

or

$P(\hat{p} \geq .15) =$ Normalcdf(.15, 1E99, .1, .0286) =

$\boxed{.0402}$

11. What is the probability that a SRS of n = 120 boys will show that at most 12% have a 3.0 average? Show all work.

$P(\hat{p} \leq .12) = P(Z \leq .73) =$

$Z = \frac{.12 - .1}{.0274} = .7299 = \boxed{.7673}$

or

$P(\hat{p} \leq .12) =$

Normalcdf(-1E99, .12, .1, .0274)

.7673