MOUNT HEBRON HIGH SCHOOL

Common Core Algebra II

Midterm Review for Regular and GT

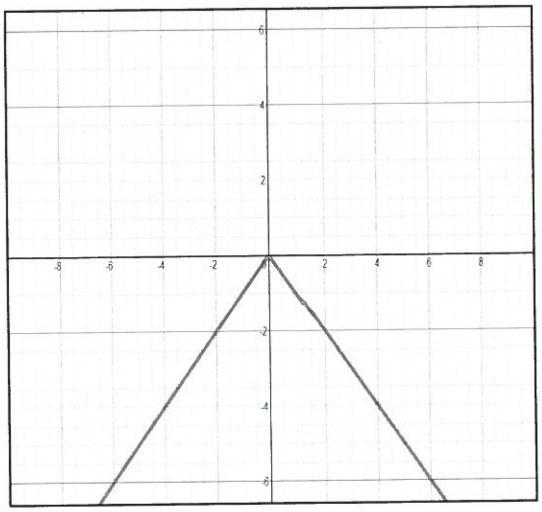
MHHS MATH DEPT – CC Alg II Team

2015 – 2016 school year

This is the midterm review packet that has examples for each topic covered in the Fall semester. These are not the exact problems you will see on the midterm. The purpose of this document is to help you practice problems in order to deepen your understanding of the concepts learned.

Unit 1: Family of Functions

Directions: For the graphs in #1 and 2, determine the type of function represented in the graph, domain, range, any increasing intervals, any decreasing intervals, symmetry, intervals of continuity and absolute or local extrema.



1.

Type of Function: absolute value

Domain: $\mathbb{R}$

Range: $(-\infty, 0]$ $y \le 0$

Intervals of Increasing: $(-\infty, 0)$

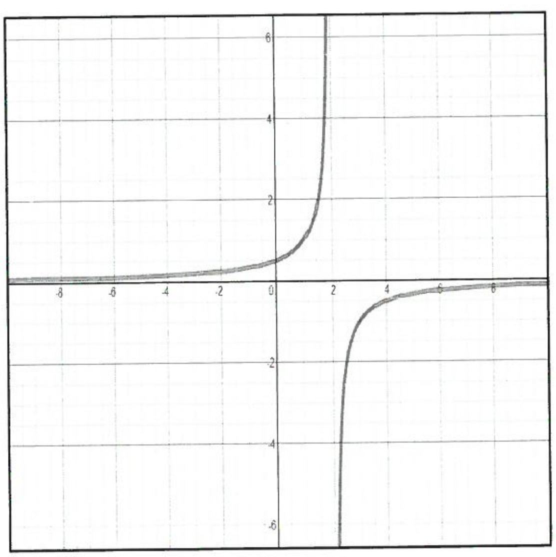
Intervals of Decreasing: $(0, \infty)$

Symmetry: even

Intervals of Continuity: $(-\infty, \infty)$

Maximum: $(0, 0)$ (Absolute or Relative)

Minimum: none (Absolute or Relative)



2.

Type of function: rational

Domain: $(-\infty, 2) \cup (2, \infty)$

Range: $(-\infty, 0) \cup (0, \infty)$

Intervals of Increasing: $(-\infty, 2)(2, \infty)$

Intervals of Decreasing: none

Symmetry: none

Intervals of Continuity: $(-\infty, 2)(2, \infty)$

Maximum: none (Absolute or Relative)

Minimum: none (Absolute or Relative)

3. **Graph $y = x^3 - 4x^2 + x - 1$ in your calculator and find the following:**

Type of Function: Cubic

Domain: $\mathbb{R}$

Range: $\mathbb{R}$

Intervals of Increasing: $(-\infty, .13)$ $(2.54, \infty)$

Intervals of Decreasing: $(.13, 2.54)$

Symmetry: none

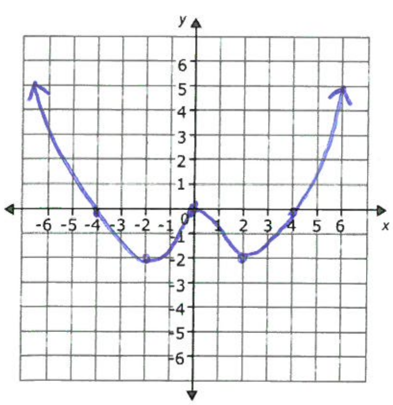
Intervals of Continuity: $(-\infty, \infty)$

Maximum: $(.13, -.94)$ (Absolute or Relative)

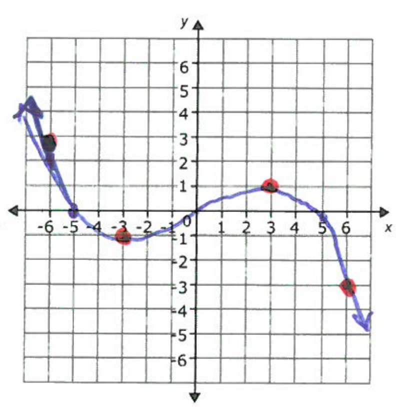
Minimum: $(2.54, -7.88)$ (Absolute or Relative)

4. **Sketch an example of each type of symmetry:**

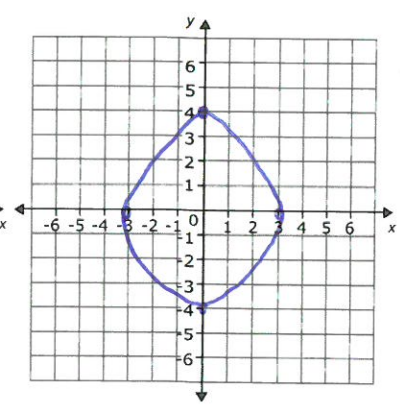
Even



Odd

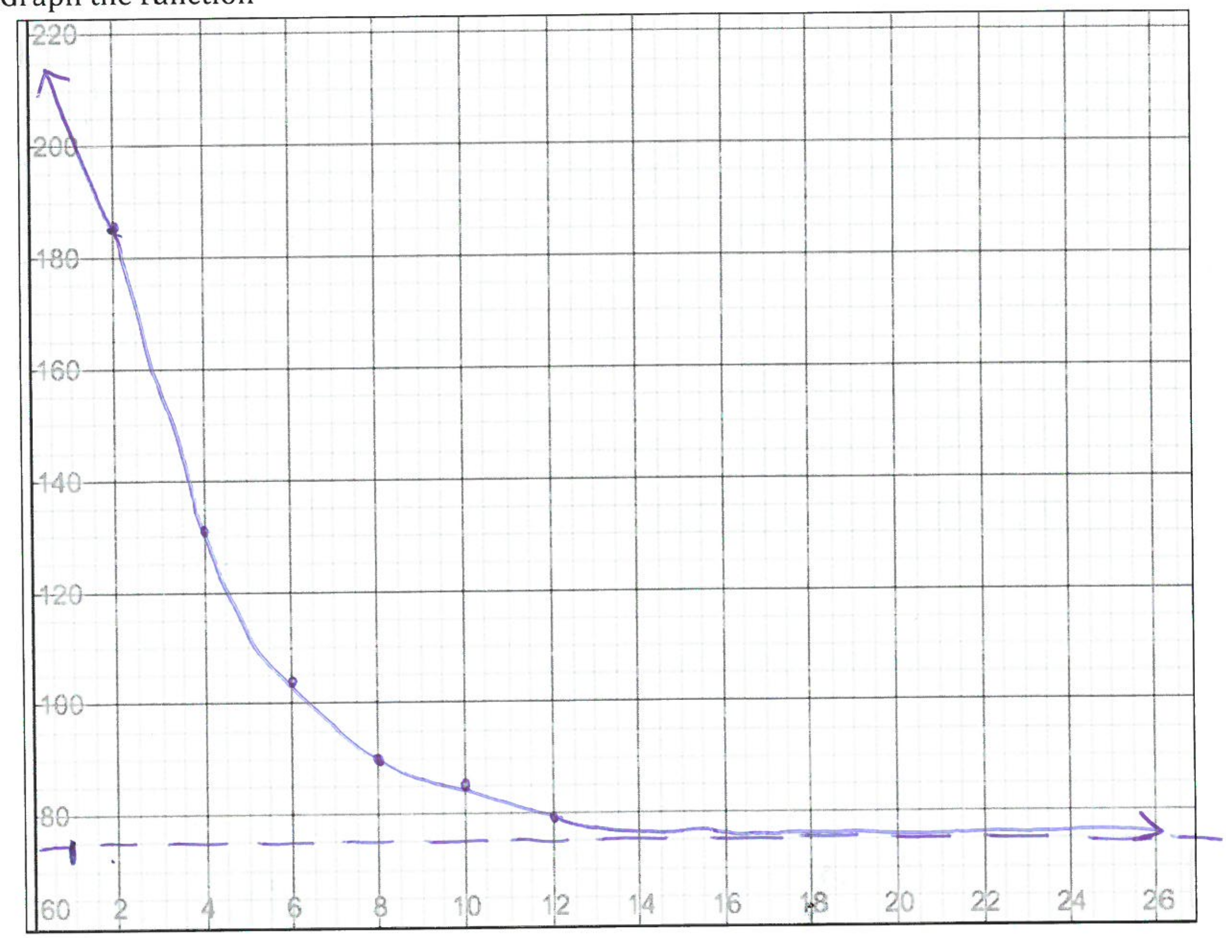


Both



5. Let $y = 212(.72)^x + 75$ model the temperature of a cup of tea in Celsius, after x minutes.

Graph the function



a. Will the inverse be a function? How can you tell by looking at the graph of the original function? Yes, since this function passes the horizontal line test, its inverse will be a function

b. What is an easy way to find the domain and range of the inverse? Find the domain and range. Switch them

original	Inverse
D: $\mathbb{R}$	D: $(75, \infty)$
R: $(75, \infty)$	R: $\mathbb{R}$

c. What type of symmetry is there between a function and its inverse?

$y = x$

6. Find the inverse for each of the following functions.

a. $y = \frac{3}{5}x - 2$	b. $y = \sqrt{x-3} + 4$	c. $y = \log_5(x-1) + 2$
$x = \frac{3}{5}y - 2$	$x - 4 = \sqrt{y-3}$	$x - 2 = \log_5(y-1)$
$x + 2 = \frac{3}{5}y$	$(x-4)^2 = y - 3$	$5^{(x-2)} = y - 1$
$\frac{5}{3}(x+2) = y^{-1}$	$(x-4)^2 + 3 = y^{-1}$	$5^{(x-2)} + 1 = y^{-1}$
	D: $x \geq 4$	

Directions: For #7 and 8, use the graphing calculator to construct a scatterplot of the data. Then use regression to calculate the equation of the function that bests models the data. Be sure to consider the scatterplot, the coefficient of determination (R^2) and the residuals plot in determining the best model.

7. A study was done to compare the speed x (in miles per hour) with the mileage y (in miles per gallon) of an automobile. The results are shown in the table. (Source: Federal Highway Administration)

x = Speed (mph)
y = mileage (mpg)

Speed	Mileage
15	22.3
22	25.5
25	27.5
30	29
35	28.8
40	30
45	29.9
50	30.2
55	30.4
60	28.8
65	27.4
70	25.3
75	23.3

scatterplot appears quadratic

$\boxed{y = -.01x^2 + .77x + 1297}$

$r^2 = .9743$

↓

strong

resids are random

8. In 1985, there were 285 cell phone subscribers in the small town of Centerville. The number of subscribers **increased** by 75% per year after 1985. How many cell phone subscribers were in Centerville in 1994?
(Note: Do not consider a fractional part of a person.)

x = years since 1985 y = cell phone subscribers

Years	1 1986	1987 2	1988 3	1989 4	1990 5	1991 6	1992 7	1993 8	1994 9
#cell phone users	498	872	1527	2672	4677	8186	14325	25069	43871

scatterplot appears exponential

$y = 284.67(1.75)^x$ Pattern

$r^2 = .9999 \approx 1$

$y = 284.67(1.75)^9 \approx 43821.19$

9. Given the table that represent the functions, f(x) and g(x), find the following...

x	-4	-3	-2	-1	0	1	2	3	4
f(x)	11	4	-1	-4	-5	-4	-1	4	11
g(x)	-11	-8	-5	-2	1	4	7	10	13

A. $f+g\,(2)$ $= -1+7 = 6$

B. $f-g\,(-3)$ $= 4 - -8 = 12$

C. $fg\,(2)$ $-1 \cdot 7 = -7$

D. $\frac{f}{g}(4)$ $\frac{11}{13}$

E. $f(g(-1))$ $g(-1) = -2$ $f(-2) = -1$

10. Given $f(x) = x^2$ and $g(x) = 2x - 3$, find

A. $f(g(1))$ $g(1) = 2(1) - 3 = -1$ $f(-1) = (-1)^2 = \boxed{1}$

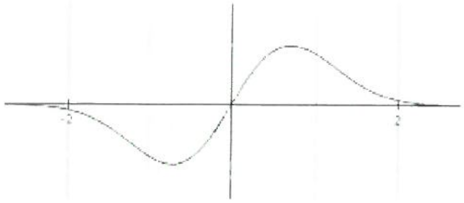
B. $g(f(-2))$ $f(-2) = (-2)^2 = 4$ $g(4) = 2(4) - 3 = \boxed{5}$

C. $f(g(x))$ $f(g(x)) = (2x-3)^2 = \boxed{4x^2 - 12x + 9}$

D. $g(f(x))$

$g(f(x)) = 2(x)^2 - 3 = \boxed{2x^2 - 3}$

11._Determine if the function given is even, odd or neither.



a. odd

b. $f(x) = x^3 - 5x^2$ $f(2) = 8 - 20 = -16$ Neither

$f(-2) = -8 - 20 = -28$

c.

X	-2	-1	0	1	2
f(x)	5	3	0	-3	-5

Odd

12._Describe the transformation from the parent function, $f(x) = x^3$ to the following...

a. $f(x) = (x-1)^3 - 5$ right 1 down 5

b. $f(x) = -x^3 + 1$ up 1 reflected over $y=0$ (x-axis)

c. $f(x) = \frac{1}{2}(x+2)^3 + 4$ left 2 up4 compressed by a factor of $\frac{1}{2}$

Unit 2: Geometric Sequences, Series, Exponents and Logarithms

13. Find the explicit equation for the n^{th} term of the geometric sequence 3, -6, 12, -24, ...

$a_n = 3(-2)^{(n-1)}$

14. Find the sum of the first 8 terms of the geometric sequence 1, 2, 4, 8, ...

$S_n = \frac{1(1-2^n)}{1-2} = \frac{1-2^8}{-1} = \boxed{255}$

15. Find the sum of the infinite series $5 + \frac{10}{3} + \frac{20}{9} + \frac{40}{27} + \ldots$ $r = \frac{2}{3}$ $\frac{5}{1-2/3} = \frac{5}{\frac{1}{3}} = \boxed{15}$

$\frac{a}{1-r}$

16. Find the value of $\sum_{n=5}^{8} 2n-1$

$2(8)-1 + 2(7)-1 + 2(6)-1 + 2(5)-1 = \boxed{48}$

17. Does the series diverge or converge? If possible, evaluate the sum. $1 - \frac{1}{4} + \frac{1}{16} - \ldots$

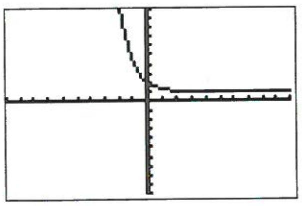
$r = \frac{-1}{4}$ the series converges $\frac{1}{1 - \frac{-1}{4}} = \frac{1}{5/4} = \boxed{\frac{4}{5}}$

Directions: For #18-21 Determine whether the function is an exponential growth or decay.

18. $y = 1.2(2)^x$ growth

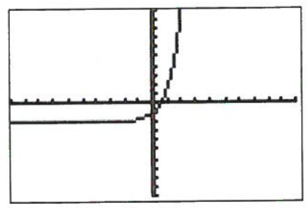
19. $y = 8(.95)^x$ decay

20.



decay

21.



growth

22. A science experiment involves periodically measuring the number of mold cells present on a piece of bread. At the start of the experiment, there are 50 mold cells. Each time a periodic observation is made, the number of mold cells triples.

a. Write a function formula equation ($y = \cdots$) for the number of mold cells present, where x stands for the observation number.

$y = 50(3)^x$

b. Fill in the missing outputs of this table.

x = observation number	0	1	2	3	4	5
y = mold cell count	50	150	450	1350	4050	12150

c. Suppose that the mold begins to be visible as green coloration when the mold cell count exceeds 100,000. On which observation will this happen?

On the 7th observation they will see mold, though technically it occurs between the 6th + 7th

d. What will be the mold cell count on the 10th observation?

$50(3)^{10} = 2,952,450$

23. Julie gets a pre-paid cell phone. Initially she has a $40.00 balance on the phone. Each minute of talking costs $0.15. Let x stand for the amount of time in minutes that Julie has talked on the phone, and let $f(x)$ stand for the remaining dollar value of the phone.

a. Is $f(x)$ a linear function or an exponential function? Explain how you know.

f(x) is a linear function because there is a constant rate of decrease

b. Find a function formula equation $f(x) =$ $40 - .15x$

c. Find the value of $f(0)$ and explain its meaning in terms of the cell phone.

$f(0)=40$

When Julie gets the phone + before she makes any calls, she has $40 on it

d. Find the value of $f(100)$ and explain its meaning in terms of the cell phone.

$f(100) = 40 - .15(100) = 25$

When Julie has talked for 100 min., she has $25 remaining on the phone

e. Find the value of x that makes $f(x) = 10$, and explain its meaning in terms of the cell phone.

$40 - .15x = 10$

$-.15x = -30$

$x = 200$

At 200 minutes, Julie will have $10 on her cell phone

f. Find the value of x that makes $f(x) = 0$, and explain its meaning in terms of the cell phone.

$40 - .15x = 0$

$-.15x = -40$

$x = 266.67$

At 266 2/3 of a minute, Julie runs out of time

24. You have inherited land that was purchased for $30,000 in 1960. The value of the land increased by, approximately, 5% per year. What is the approximate value of the land in the year 2015?

$y = 30,000(1.05)^x$

$y = 30,000(1.05)^{55} = \$439,068.93$

25. James deposited $2,500 in a bank account. Find the balance after 5 years for each of the following situations:

$A = 2500\left(1 + \frac{.025}{12}\right)^{5 \cdot 12}$

a. The account pays 2.5% annual interest compounded monthly. $2832.51

b. The account pays 1.75% annual interest compounded continuously.

$A = 2500(e)^{(5 \cdot .0175)} = \$2728.61$

26. For each function below:

a) State whether the function is growth or decay.

b) Identify the y-intercept.

c) State how the graph has shifted.

d) Identify the domain and range.

e) Write the equation of the asymptote.

f) Graph.

i. $o(x) = 2^{x-3}$

a. growth

b. (0, .125)

c. right 3

d. D: ℝ R: y > 0

e. y = 0

ii. $r(x) = \frac{1}{2}(1.4)^x$

a. growth

b. (0, 1/2)

c. compressed by $\frac{1}{2}$

d. D: ℝ R: y > 0

e. y = 0

iii. $g(x) = 3(.60)^x - 2$

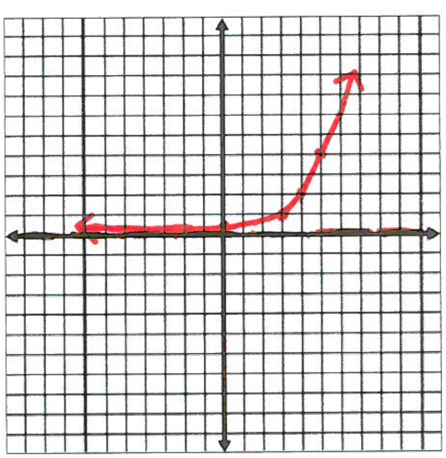
a. decay

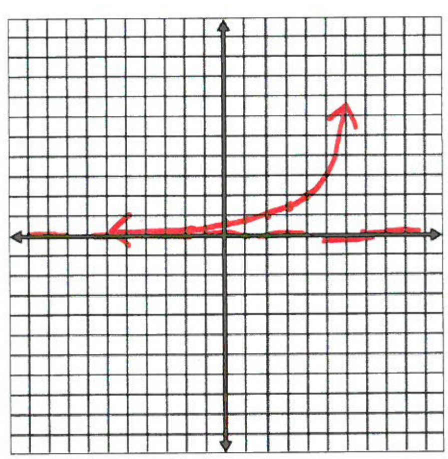
b. (0, 1)

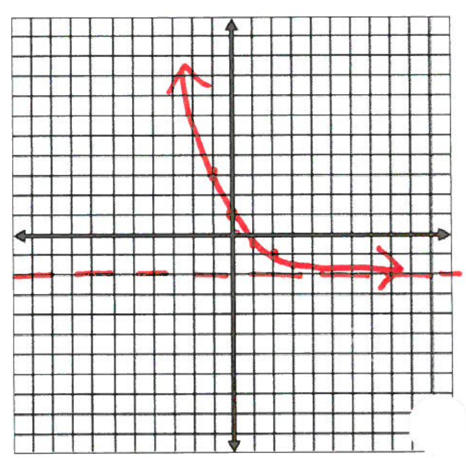
c. down 2, stretched by 3

d. D: ℝ R: y > −2

e. y = −2







27. For each function below:

a) Identify the domain

b) Identify the range

c) State how the graph has shifted.

d) Write the equation of the asymptote.

e) State the intercepts.

f) Graph.

i. $f(x) = \log_2(x+4)$

a. $x > -4$

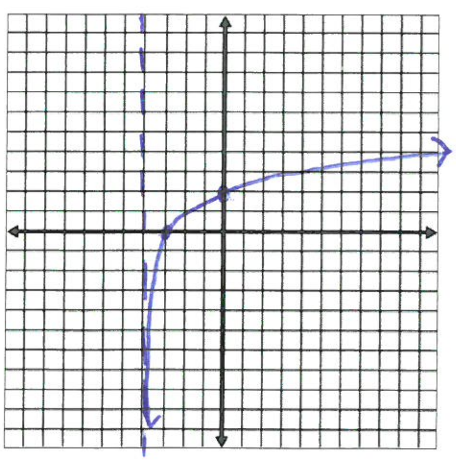
b. $\mathbb{R}$

c. left 4

d. $x = -4$

e. $(0,2)$ $(-3,0)$

$2^y = x+4$



ii. $f(x) = 3 + \log_5 x$

a. $x > 0$

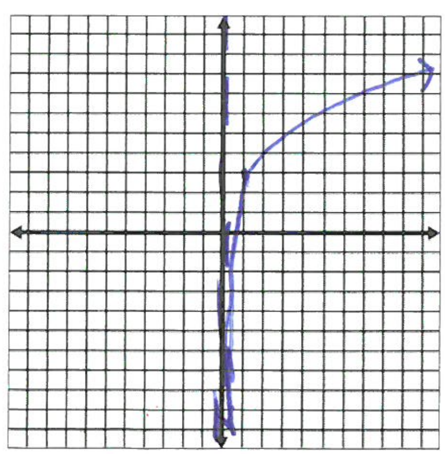
b. $\mathbb{R}$

c. up 3

d. $x = 0$

e. no y-int $(.008, 0)$

$(1,3)$ $5^{y-3} = x$



iii. $f(x) = \log(x-2) + 6$

a. $x > 2$

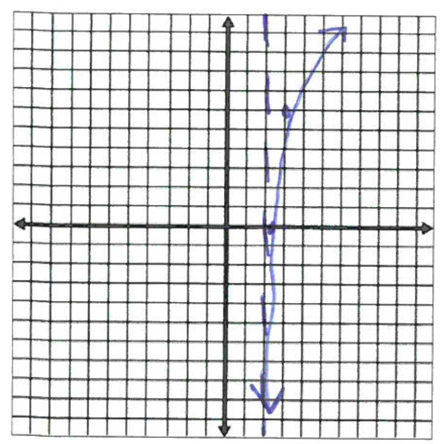
b. $\mathbb{R}$

c. right 2, up 6

d. $x = 2$

e. no y-int $(\sim 2, 0)$

$(3,6)$ $10^{y-6} = x-2$



28. Evaluate $\log_2 32$ = 5

29. Write as a single logarithm: $\log x + 7\log y - \log 6$

$\log \frac{xy^7}{6}$

30. (GT only) Expand the following logarithm: $\log\left(\frac{4x}{x+9}\right)^2$

$2\log 4 + 2\log x - 2\log(x+9)$ -or- $2[\log 4 + \log x - \log(x+9)]$

31. Solve for x: $\log_4(5x-1) = 3$

$4^3 = 5x - 1$

$64 + 1 = 5x$

$65 = 5x$

$x = 13$

32. Solve for x: $3 + \log x^2 = 7 + \log x$

$\log_{10} 10^3 + \log x^2 = \log_{10} 10^7 + \log x$

$\log 1000x^2 = \log 10{,}000{,}000x$

$1000x^2 = 10{,}000{,}000x$

$\boxed{x = 10{,}000}$

33. (GT only) Solve for x: $4^{3x} = 64^{(2x+5)}$

$4^{3x} = (4^3)^{(2x+5)}$

$4^{3x} = 4^{6x+15}$

$3x = 6x + 15$

$-3x = 15$

$\boxed{x = -5}$

Directions: For #34 and 35, use a calculator to approximate each to the nearest hundredth.

34. ln 6.2

1.82

35. e^{10} 22026.47

36. Expand the logarithm: $\ln\frac{a^2}{b}$ $2\ln a - \ln b$

37. Write as a single logarithm and simplify: $\ln 10 - 5\ln 2$

$\ln 10 - \ln 2^5 = \ln 10 - \ln 32 = \ln\left(\frac{10}{32}\right) = \ln\left(\frac{5}{16}\right)$

38. Solve for x (round to the nearest thousandth): $e^{x+2} = 59$

$\ln 59 = x + 2$

$-2 + \ln 59 = x$

$2.078 = x$

39. Solve for x: $\ln(-5n) = \ln(4-3n)$

$-5n = 4 - 3n$

$-2n = 4$

$n = -2$

$\boxed{\text{no solution}}$

40. Solve for x (round to the nearest thousandth): $\ln 5 + \ln(4-5x) = 3$

$4 - 5(-.0034) = 4.017$

$\ln(20-25x) = 3$

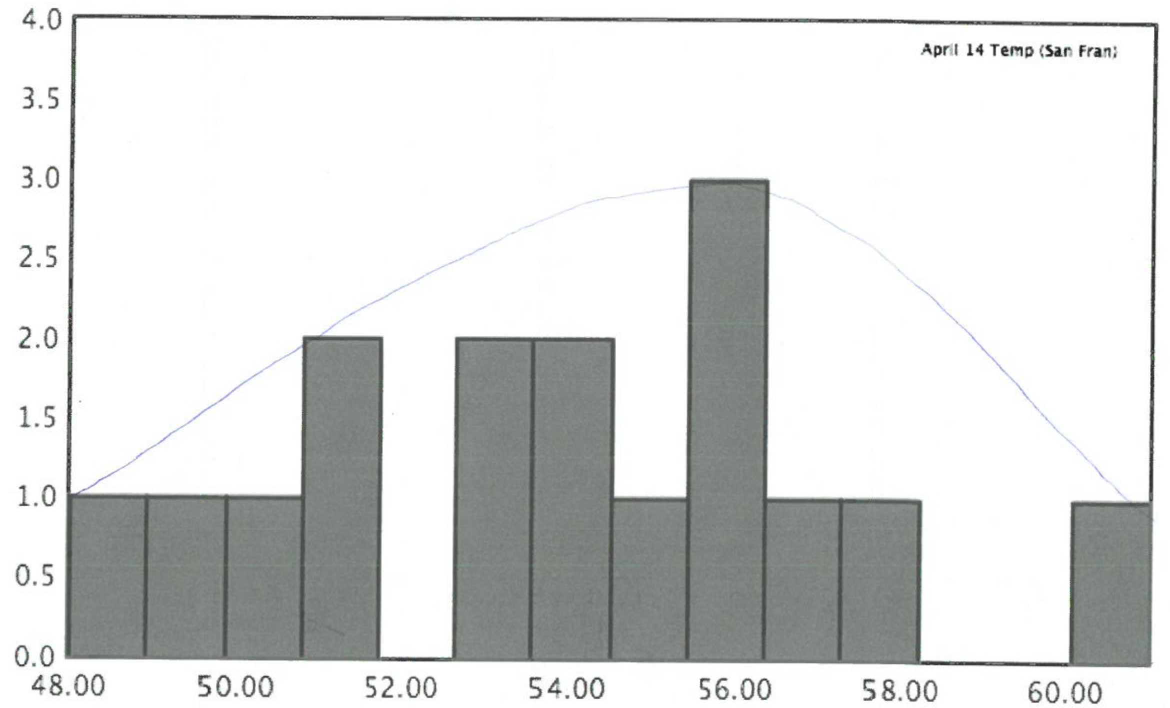
$e^3 = 20 - 25x$

$\boxed{x = -.0034}$

Unit 3: Statistics

41. The temperature in San Francisco on April 14 has been recorded for the last 16 years.

Describe the shape of the distribution in context. Then choose an appropriate measure of center and an appropriate measure of spread and calculate them for this sample. Be sure to report them in context.



Shape - roughly symmetric

mean - $\bar{x} = 53.81$

std dev - = 3.37

med = 54

IQR = 5

Some might say that this is skewed slightly to the left. If so, they should only refer to median and IQR.

42. A school district is trying to determine if library assistants are necessary in schools. So they decide to take a survey and have several possible sampling methods for taking this survey. Identify the name of each sampling method and how it might lead to bias in the survey, if it does. Identify the sampling method that gives the best chance for a representative sample.

A. Randomly select 3 schools and survey everyone in the school. $\boxed{\text{Cluster sample}}$

unless the 3 schools are representative, there will be bias

B. Get a random sample of 50 people in an elementary school, 50 people in a middle school and 50 people in a high school. $\boxed{\text{Stratified random sample}}$

If the proportions of hs, ms, and elementary students are not similar, there can be undercoverage

C. Post the survey on the school district website and collect responses for a week.

$\boxed{\text{voluntary response}}$ only those who care respond

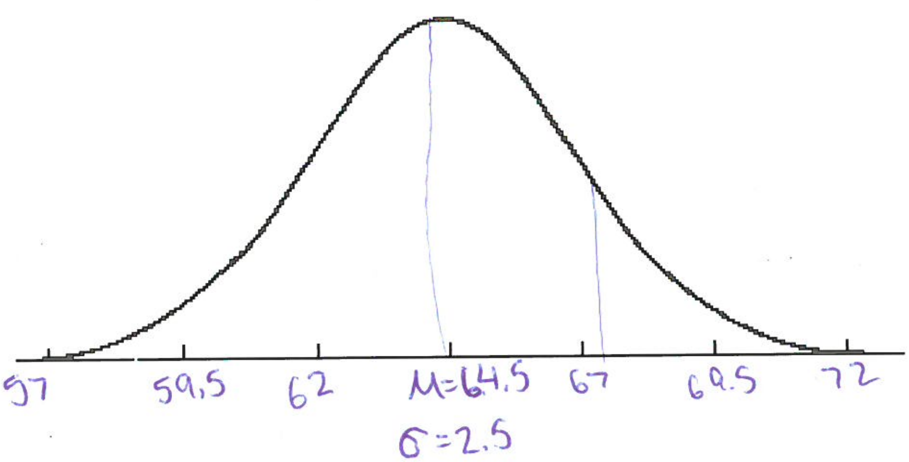
D. Select a random sample of 500 students and employees in the school district.

$\boxed{\text{SRS}}$ Best chance for a representative sample

43. The mean height of college-aged women is 64.5 in with a standard deviation of 2.5 in.

Part A

Draw a normal curve and label the mean and points one, two, and three standard deviations above and below the mean.



Part B

A. About What percent of college-aged women are taller than 67 inches? ~16%

B. Between what heights do the middle 95% of college-aged women fall? 59.5 - 69.5

C. What percent of college-aged women are shorter than 59.5 inches?

2.5%

44. When the school district's survey was complete they found that 52% of the 100 people sampled believe that library assistants are necessary. The school district had believed that only 35% of the population believes that library assistants are necessary. They are going to run a hypothesis test to see if the sample results are significant evidence that they might be wrong.

Write the null and alternate hypotheses for this test.

p = proportion of people who think library assistants are necessary.

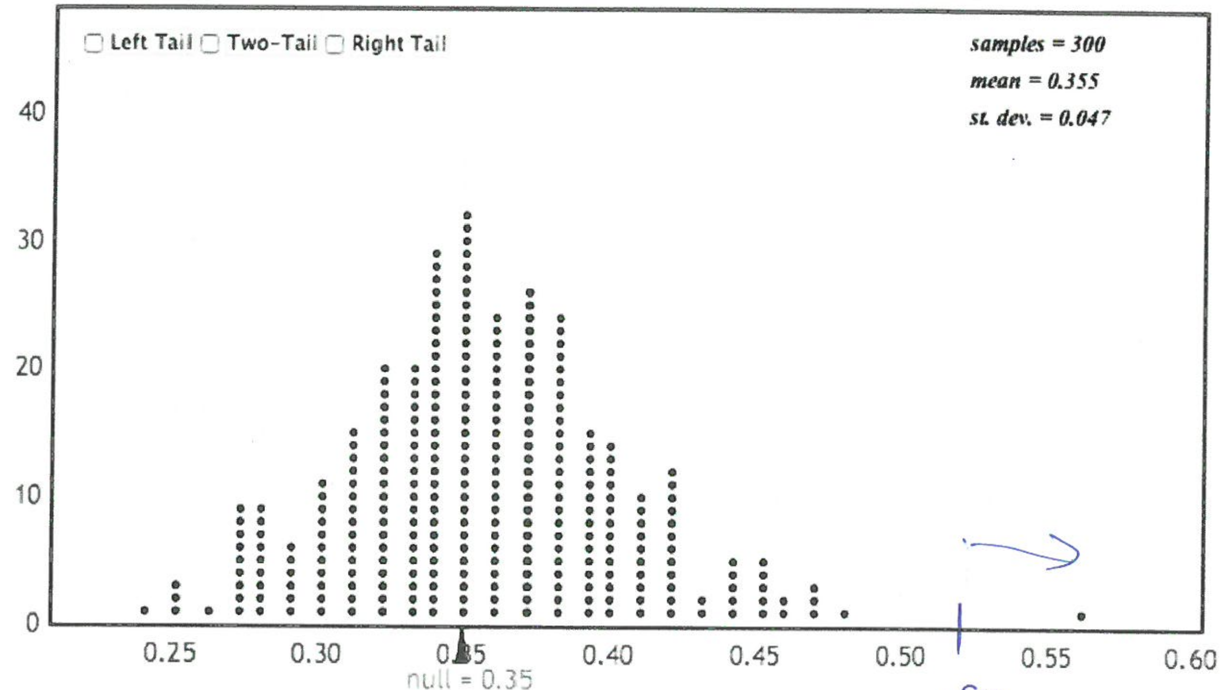
H_0: $p = .35$

H_A: $p > .35$

Sample → $\hat{p} = .52$

When they ran the test they took 300 samples and the results are on the next page.

Calculate the P-value for this test. Interpret the results in the context of the situation.



$\text{P-Value} = P(\hat{p} > .52) = \frac{1}{300} = .003$

very surprising

If the proportion of the school district that feels library assistants are necessary is .35 then it is very surprising to get a sample as high as .52. This only happens about .3% of the time. This is very strong evidence against the null.

45. A couple was planning to move from Atlanta to St Louis because they thought they could reduce the amount of time they spent commuting to and from work. So they selected random sample of people who drive to work in Atlanta and a random sample of people who drive to work in St. Louis and found that the commuters in Atlanta spend a mean amount of 4 minutes longer commuting than the commuters in St Louis.

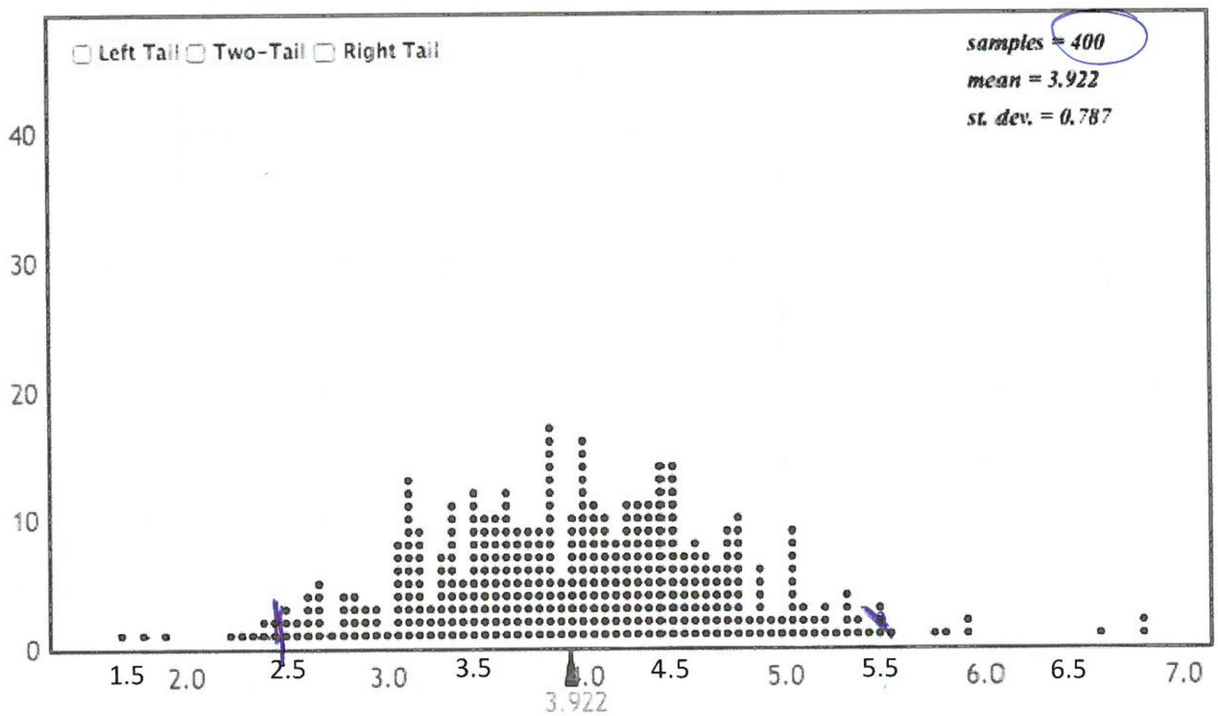
μ_A = Atlanta's mean time
μ_S = St Louis's mean time
$\bar{x}_A - \bar{x}_S = 4$ ← Sample data

Determine the null and alternate hypotheses.

H_0: $\mu_A - \mu_S = 0$

H_A: $\mu_A - \mu_S > 0$

The results of a simulation of 400 trials are shown in the dotplot.



Construct a 95% confidence interval for the difference in commute times in Atlanta and St. Louis. 95% of 400 = 380
10 in each tail

(2.5, 5.5)

Interpret the interval. We are 95% sure that the mean commute time for Atlanta is from 2.5 minutes longer than St Louis to 5.5 minutes longer than St. Louis.

Does the interval offer evidence against the null hypothesis? Why or why not?

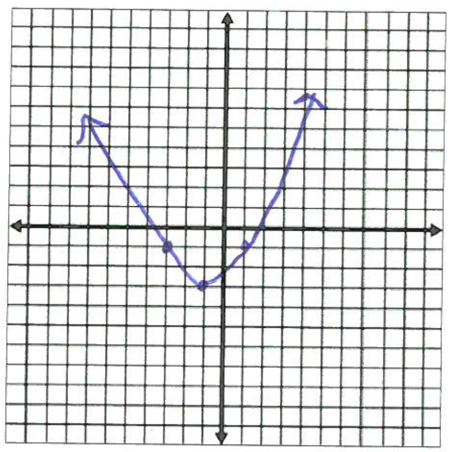
Since 0 is not in the interval, we have strong evidence against the null hypothesis.

Unit 4: Quadratics

46. For $y = \frac{1}{2}(x+1)^2 - 3$, identify the:

a. Vertex: $(-1, -3)$
b. Max/min: abs min $(-1,-3)$
c. Domain: $\mathbb{R}$
d. Range: $[-3, \infty)$
e. Axis of Symmetry: $x = -1$
f. Transformations: rt 1, down 1, compressed by $\frac{1}{2}$

g. Graph:



47. For $y = x^2 + 8x + 5$, convert to vertex form then identify the:

$y = x^2 + 8x + \quad + 5$

$\left(\frac{8}{2}\right)^2 = 4^2 = 16$

$y = x^2 + 8x + 16 + 5 - 16$

$y = (x+4)^2 - 11$

a. Vertex: $(-4, -11)$
b. Max/min: min $(-4, -11)$
c. Domain: $\mathbb{R}$
d. Range: $[-11, \infty)$
e. Axis of Symmetry: $x = -4$
f. Transformations: left 4 down 11

g. Graph: