Name _____ AP Stats

Date _____ Review for Ch 16-17

Random Variables and Probability Models

1. The tables below show part of the probability distribution for random variables X and Y. If X and Y are independent and the joint probability P(X = 3 , Y = 4) = 1/16 , then P (Y = 1) = ?

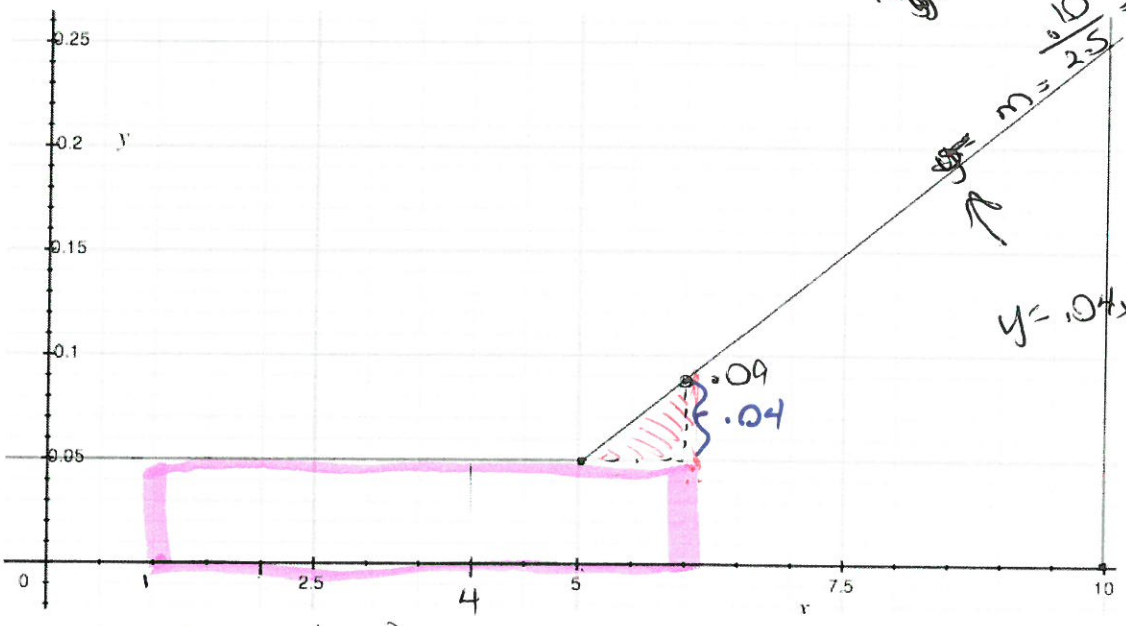
x	P (X = x)
1	1/6
2	2/3
3	? $\frac{1}{6}$

y	P (Y = y)
1	? $\frac{1}{6}$
2	1/4
3	1/4
4	? $\frac{3}{6}$

$\left(\frac{1}{6}\right)(P(4)) = \frac{1}{16}$

$P(4) = \frac{6}{16} = \frac{3}{8}$

2. Here is the probability distribution function for a continuous random variable X. Determine the following probabilities.



.2 a. P (X ≤ 4) 4(.05)

.1 b. P (2 ≤ X ≤ 4) 2(.05)

0 c. P (X = 3) $\frac{1}{2}(.5)(.09) = .0225$ .27

~~.295~~ .27 d. P (1 ≤ X ≤ 6) $\frac{1}{2}(b)(h)$

~~.295~~ .27 e. P (1 < X < 6)

$.05 = .04(2) + b$

$.05 = .04(5) + b$

$.05 = .2 + b$

$-.15 = b$

$y = .04(6) - .15$

$y = .24 - .15 = .09$

Name _____ AP Stats

Date _____ Review for Ch 16-17

3. Consider the probability distribution X for a lopsided die.

Face	1	2	3	4	5	6
P(x)	.12	.20	.17	.19	.15	.17

a. Verify that this a probability distribution.

$.12 + .20 + .17 + .19 + .15 + .17 = 1.0$

b. What is the expected value of X?

$\mu_x = (.12)1 + 2(.20) + 3(.17) + 4(.19) + 5(.15) + 6(.17) = 3.56$

c. What is the variance of X?

$\sigma_x^2 = (1-3.56)^2(.12) + (2-3.56)^2(.2) + \ldots + (6-3.56)^2(.17) = 2.6864$

d. What is the standard deviation of X?

$\sigma_x = \sqrt{2.6864} = 1.6390$

e. What is μ_{3x}?

$\mu_{3x} = 3\mu_x = 3(3.56) = 10.68$

f. What is the variance and standard deviation of $4+3X$?

$\sigma^2_{3x+4} = 9(2.6864) = 24.1776$

$\sigma_{3x+4} = 4.9171$

g. What is the standard deviation of $5X$?

$\sigma_{5x} = 8.195$

4. Shaquille O'Neal is about a 45% free throw shooter (he makes 45% of his free throws). Assume that each shot is independent of the next. How many points, on average, is he expected to score if he shoots a one-and-one? [A one-and-one means that he takes a second shot if and only if he makes the first]

Show all work.

~~$\mu = np = .45(2) = .9$~~

X = # pts	0	1	2
	.55	(.45)(.55)	$(.45)^2$

$\mu_x = 0(.55) + 1(.2475) + 2(.2025) = .6525$

Name _____ AP Stats

Date _____ Review for Ch 16-17

5. Random Variable X has 23 values with a mean of 23.4 and a standard deviation of 2.5. Random Variable Y has 23 values with a mean of 18.2 and a variance of 1.25. Find the following showing any appropriate work:

a. μ_{X+Y}

$\mu_{X+Y} = 41.6$

b. Find μ_{X+Y}, if X and Y are independent.

$\mu_{X+Y} = 41.6$

c. Find σ^2_{X+Y}

$\sigma^2_{X+Y} = (2.5)^2 + (1.25)$

7.5

d. Find σ^2_{2X-Y}

$\sigma^2_{2X-Y} = 4(2.5)^2 + 1.25$

26.25

e. Find σ_{2X+3Y}

$\sigma_{2X+3Y} = \sqrt{4(2.5)^2 + 9(1.25)}$

6.0208

f. Find μ_{2X+3Y}

$\mu_{2X+3Y} = 2(23.4) + 3(18.2)$

101.4

6. Suppose the average height of policemen is 71 inches with a standard deviation of 4 inches, while the average for policewomen is 66 inches, with a standard deviation of 3 inches. If a committee looks at all ways of pairing up one male with one female officer, what will be mean and standard deviation for the difference in heights for the set of possible partners? Show work.

$\mu_M = 71 \quad \sigma_M = 4$

$\mu_W = 66 \quad \sigma_W = 3$

$\mu_{M-W} = 71 - 66 = 5$

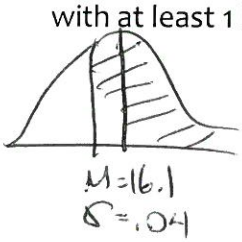
$\sigma_{M+W} = \sqrt{4^2 + 3^3} = 5$

Name _____
Date _____

AP Stats
Review for Ch 16-17

7. A packing machine is set to fill a cardboard box with a mean of 16.1 ounces of cereal. Suppose the amounts per box form a normal distribution with a standard deviation equal to 0.04 ounce. Make a sketch and answer each of the following.

a. What percent of boxes will end up with at least 1 lb of cereal?



$P(x \geq 16) =$

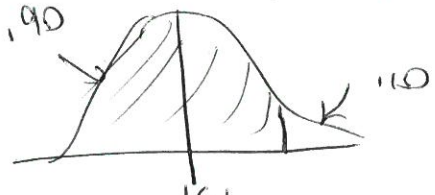
$z = \frac{16.0 - 16.1}{.04}$

$z = -2.5$

$P(z \geq -2.5) =$

$1 - P(z \leq -2.5) = 1 - .0062 = .9938$

b. Ten percent of the boxes will contain more than how many ounces?

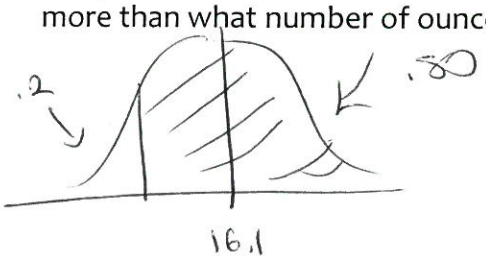


$z = 1.28$

$1.28 = \frac{x - 16.1}{.04}$

$x = 16.1512$

c. Eighty percent of the boxes will contain more than what number of ounces?

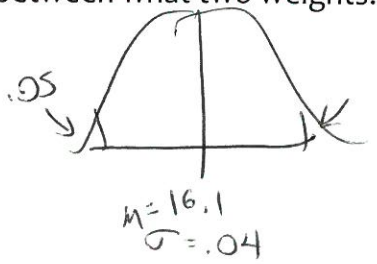


$z = -.84$

$-.84 = \frac{x - 16.1}{.04}$

$x = 16.0664$

d. The middle 90% of the boxes will be between what two weights?



$z = +1.64$

$z = -1.64$

$-1.64 = \frac{x - 16.1}{.04}$ $1.64 = \frac{x - 16.1}{.04}$

16.0344 and 16.1656

8. Each child born to a particular set of parents has a probability 0.25 of having blood type O. Suppose these parents have 4 children and Y is the random variable that counts the number of children that have type O blood. Give the probability distribution of Y in the space below. Find the expected number of children with type O blood. Find the variance and the standard deviation of the number of children out of the 4 that have type O blood (include units in your answer). $p = .25$ $n = 4$

x	0	1	2	3	4
P	.3164	.4219	.2109	.0469	.0039

$\mu_x = 1$ $np = .25(4) = 1$

$\sigma_x = \sqrt{.75} = .8660$ $\sigma = \sqrt{npq} = .8660$

Name _____ AP Stats

Date _____ Review for Ch 16-17

9. A friend of yours is a 75% free-throw shooter, that is, he will make 75% of the shots he takes. If he shoots 5 shots and X represents the number of made shots, answer the following. $P = .75$

a. Is X binomial or geometric? Justify your answer.

We are counting successes

b. Give the Probability Distribution Function (pdf) of X, using the "Vars" menu of your calculator.

X	0	1	2	3	4	5
P	.001	.0146	.0879	.2637	.3955	.2373

c. Use the Binomial Probability Formula, showing all work, to find the probability that he will make 3 shots in the 5 taken. ${}_5C_3$

$$P(x=3) = \frac{5!}{3!\,2!}(.75)^3(.25)^3 = .2637$$

d. Find the mean and standard deviation of X.

$$M_x = 5(.75) = 3.75$$

$$\sigma_x = \sqrt{npq} = \sqrt{5(.75)(.25)} = .9682$$

e. Find the probability that of the 5 throws, he will make less than 3.

$$P(x<3) = P(x \le 2) \quad \text{Binomcdf}(5, .75, 2) = .1035$$

f. Find the probability that of the 5 throws, he will make at least 3.

$$P(x \ge 3) = 1 - P(x \le 2) = 1 - .1035 = .8965$$

g. Find the probability that of the 5 throws, he will make between 1 and 4.

$$P(1<x<4) = P(x=2) + P(x=3) = .3516$$

h. In how many ways can he make 2 of his 5 free throws?

$${}_5C_2 = \frac{5!}{2!\,3!} = 10$$

i. Let y be the random variable that counts the number of shots required to get the first made shot. Is Y binomial or geometric?

j. Find the probability that it takes 3 shots to make the first shot.

$$P(x=3) = (.25)^2(.75) = .0469$$

k. Find the probability that it takes more than 3 shots to make the first shot.

$$P(x>3) = 1 - P(x \le 3) = \text{Geometcdf}(.75, 3)$$

$$1 - .9844 = .0156$$

Name _____ AP Stats

Date _____ Review for Ch 16-17

1. Find the probability that he will take no more than 3 shots to make the first shot.

$P(X \leq 3) = \text{Geometcdf}(.75, 3) = .9844$

10. An unbalanced coin has P (Heads) = 3/8. Let X count the # of heads in 10 flips of the coin.

a. Is X Geometric or Binomial? Justify.

We are counting the # of success in a finite # of observations

b. Find the mean and standard deviation of X.

$\mu_x = 10\left(\frac{3}{8}\right) = 3.75$

$\sigma_x = \sqrt{10\left(\frac{3}{8}\right)\left(\frac{5}{8}\right)} = 1.5309$

c. Find P (X = 2) using the Binomial Formula only.

$P(x=2) = {}_{10}C_2\left(\frac{3}{8}\right)^2\left(\frac{5}{8}\right)^8 = \frac{10!}{2!\,8!}\left(\frac{3}{8}\right)^2\left(\frac{5}{8}\right)^8$

$45\left(\frac{3}{8}\right)^2\left(\frac{5}{8}\right)^8 = .1473$

d. Find P (X = 2) using your calculator only.

$\text{Binompdf}\left(10, \frac{3}{8}, 2\right) = .1473$

e. Find P (X ≤ 3).

$\text{Binomcdf}\left(10, \frac{3}{8}, 3\right) = .4467$

Find P (2 ≤ X < 4). $\text{Binomcdf}\left(10, \frac{3}{8}, 3\right) - \text{Binomcdf}\left(10, \frac{3}{8}, 1\right)$

$P(X=2) + P(X=3) = \text{Binompdf}\left(10, \frac{3}{8}, 2\right) + \text{Binompdf}\left(10, \frac{3}{8}, 3\right) = .3831$

11. Let Y = the number of rolls required to get a 4 on a 12-sided die.

a. Is Y binomial or geometric? Justify. $p = \frac{1}{12}$

We are counting the # of trials until we get the 1st success

b. How many rolls should we expect to make to get the first 4?

$\mu_Y = \frac{1}{\frac{1}{12}} = 12$

c. Find P (Y = 6). $P(Y=6) = \left(\frac{11}{12}\right)^5\left(\frac{1}{12}\right) = .0539$

$\text{Geometpdf}\left(\frac{1}{12}, 6\right) =$

d. Find P (Y > 5). $= 1 - P(Y \leq 4)$

$1 - \text{Geometcdf}\left(\frac{1}{12}, 4\right) = .7061$

Name _____ AP Stats

Date _____ Review for Ch 16-17

12. Amarillo Slim, a professional dart player, has an 80% chance of hitting the bull's eye on a dartboard with any throw. Suppose that he throws 10 darts, one at a time, at the dartboard.

Let X = the number of bull's eyes.

a. Is X Binomial or Geometric?

$p = .8$

$n = 10$

b. Compute the mean and variance of the number of bull's eyes in 10 throws.

$\mu = np = 8$

$\sigma = \sqrt{npq} = \sqrt{10(.8)(.2)} = 1.2649$

c. Find the probability that he hits the bull's eye at least four times.

$P(X \geq 4) = 1 - P(X \leq 3) = 1 - \text{Binomcdf}(10, .8, 3) = .9991$

d. Find $P(3 \leq X < 7)$.

$P(X \leq 6) - P(X \leq 2) = \text{Binomcdf}(10, .8, 6) - \text{Binomcdf}(10, .8, 2) = .1208$

Let's suppose that Slim keeps throwing darts until he hits the bull's eye.

Geometric

e. Find the probability that Slim's first bulls's eye occurs on the fourth throw.

$P(x = 4) = (.2)^3(.8) = .0064$

f. Find the probability that Slim's first bull's eye occurs in more than 6 throws.

$P(x > 6) = 1 - P(x \leq 6) = 1 - \text{Geometcdf}(.8, 6) = .0006 \approx .0001$

13. Luxury cars According to *infoplease*, 18.8% of the luxury cars manufactured in 2003 were silver. A large car dealership typically sells 50 luxury cars a month.

$p = .188$ $q = .812$ $n = 50$

a. Explain why you think that the luxury car sales can be considered Bernoulli trials.

- success = sell silver car failure = sell non-silver car

$p = .188$ is constant

Silver car sales are assumed to be independent

b. What is the probability that the fifth luxury car sold is the first silver one?

$P(x = 5) = (.812)^4(.188) = .0517$

c. Let X represent the number of silver luxury cars sold in a typical month. What is the probability model for X? Specify the model (name and parameters), and tell the mean and standard deviation.

$np = 9.4$ Can't use Normal

$\text{Binom}(50, .188)$

$\mu_x = np$

$\sigma_x = \sqrt{npq}$

d. Using this model, determine how likely it is that the dealership will sell 16 or more silver cars in a month.

$P(x \geq 16) = 1 - P(x \leq 15)$

$1 - \text{Binomcdf}(50, .188, 15) = .0179$